DRAWING-BOARD GEOMETRY

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By

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PREFACE

The material in this book is a highly condensed and straightforward presentation of those problems which engineering draftsmen should understand about relationships involving points, lines, planes, and surfaces.

The problems are presented in two languages: a written statement and its graphical translation. Because most students are "eye minded," emphasis is placed on the graphical presentation and the written material is condensed to essentials.

The book is frankly intended as a workbook. Its format and arrangement have been carefully planned to that end. Students should be expected and directed to mark up the text and use colored pencils on the text drawings. By such methods of study, a drawing in the text, presenting a completely solved problem, may be analyzed and broken down into its elements and into a rational order of procedure.

The material presented herewith represents, with the necessary and desirable problem assignments, a two-credit course. Two class periods of one hour and two drafting-room periods of two hours are sufficient to cover the ground in one semester.

FREDERIC G. HIGBEE, SR.

May, 1938

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Steel is essential material in engineering construction. Mathematics, chemistry, physics, and language are fundamental in engineering education. Engineering drawing is the universal language of engineers. Drawing-board geometry is the grammar of graphic language.

DRAWING-BOARD GEOMETRY

CHAPTER I

ORTHOGRAPHIC PROJECTION

1. **Functions of Engineering Draftsmen.** The work of an engineering draftsman falls under two general classifications:

A. The translation of designs and data into graphical language.

This part of the draftsman's work concerning itself with "translation" is called *engineering drawing.* In this field emphasis is placed upon the theory and practice of using lines, letters, numbers, and views arranged according to a system, to describe the shape, size, and arrangement of parts of engineering structures; the kind of material used; the manner in which this material is to be finished and fastened to adjacent parts. In addition to such description, draftsmen must know how to record and express descriptions of projects requiring the use of maps, profiles, charts, graphs, and pictorial representations of the perspective and pseudoperspective type.

A very important and necessary part of such work is skill in rendering work of quality, deftness in manipulation making for speed, a thorough understanding of methods of producing and reproducing drawings of all kinds, the ability to make neat and legible freehand sketches, and, of course, facility in reading drawings.

B. The utilization of graphical statements for the quantitative determination of values required in designing, or in the elaboration of data.

The utilization of engineering drawings as a basis for the solution of problems is known by various names such as "descriptive geometry" and "the geometry of engineering drawing." For the purpose of confining the scope of this book to the fundamental problems which occur most frequently in drafting work, this part of drafting has been called *drawing-board geometry.* In this part of his work the draftsman will find considerable overlapping with the theory of orthographic projection and auxiliary views, both of which are commonly considered a part of engineering drawing. In fact, when draftsmen complete their training it is not uncommon for them to regard all such work as drafting and to think of the whole enterprise as engineering drawing.

2. **The Importance of Engineering Drawing.** The recording and conveying of ideas have engaged the best minds of all time. In literature, in the plastic and graphic arts, in music, in dramatic art, this has long been recognized as worthy. Yet in all of these the language of expression is such that there can be no assurance that the interpretation will be that intended.

Engineers, however, have perfected a means of expression by means of which even the most complicated conceptions of imagination can be recorded and correctly interpreted. So exact and free from ambiguities has the language of engineering drawing become that it is the universal language of constructive endeavor.

There is little need to point out, then, the importance to engineers of the ability to draw, both freehand and with instruments, as well as to read drawings with sureness and dispatch. There is need, however, to illustrate and to emphasize how drawings may be used as the basis for "graphical computations" to secure information of sufficient accuracy for practical purposes, and usually with considerable saving of time. Such is the purpose of drawing-board geometry.

3. **The Qualifications of Draftsmen.** In addition to the ability to read and write the graphic language, a draftsman must possess or acquire two important mental attributes which are essential characteristics.

One of these is the ability to analyze an engineering problem into its fundamentals and to state these fundamentals in a graphical form which will enable him to solve the problem and record the steps in the solution graphically.

The other is to be able to visualize the machine, structure, or object represented by the views and graphical symbols on the drawing and to translate into views, lines, and graphic symbols ideas which are creations of the imagination.

This power of analysis and the ability to visualize, coupled with facility and skill in drawing, anticipate a high order of intelligence, and leads to the conclusion that those who excel in engineering drawing may be expected to succeed in the engineering profession.

4. **The Representation of Data.** In any language there must be words and a system of organizing them into sentences and paragraphs; symbols such as letters and punctuation marks, and an established plan of putting these together so as to have meaning. In engineering drawing, points, lines, planes, and other units of graphical significance have become established as having meaning. By learning the symbolic vocabulary in common use, and the grammar by which these symbols are organized into representations having established meaning, the engineer provides himself with a graphical language without which it would not be possible to record and transfer information of constructive or factual character,

The approach to the study of this method of representation, therefore, should be that

FIGURE 1a.

An architect's rendering of a bridge. This perspective was made to show what the bridge would look like when completed. The picture was made from design drawings.

a language is being mastered, that the elements of this language—such as points, lines, planes, and surfaces familiar to the student from his study of geometry—are about to take on new and important additional meaning; and above all else that the description of the elements of structures, such as points, lines, planes, and surfaces, is complete only when such graphical representation establishes a complete statement of relationship involving three dimensions.

That such is the case may be more readily comprehended when it is remembered that all the shapes and forms dealt with in engineering drawing are of three-dimensional character and that, except for the graphical description of factual data in the form of charts and graphs, the elements of length, width, and depth must be considered and described. The problem of description involved is one of representing three-dimensional objects on a flat sheet of drawing paper having but two dimensions. An arrangement of graphical symbols must be devised to describe length, width, and depth on the plane surface of the drawing paper. Moreover, this arrangement must have fixed geometrical relationship so that, by graphic manipulation, values not shown in true size may be discovered.

All this has been made possible by a system known as orthographic projection.

5. **Shape Description.** The two known methods of representing an object of three dimensions is (1) by means of a picture, and (2) by means of what is commonly called a drawing.

Pictorial representation is known as sceneographic projection or perspective drawing, or is of the type known as pseudoperspective. Engineers use both perspective and pseudoperspectives, such as isometric, oblique, and cabinet drawing, to *illustrate* rather than to *describe*. In all pictorial

FIGURE 1*b*

A photograph of the bridge when completed. How successfully the pictorial representation accomplished its purpose may be judged by a comparison of the two pictures.

drawing the attempt is made to show how the object appears when complete, and, since such types of pictorial representations are limited to the appearance of the object from *one point of view*, they are inadequate for the exact and complete description required for design and construction purposes.

To be able to draw and to sketch by pictorial methods is an important and valuable asset to an engineer, and training in this is usually a part of the work in engineering drawing. A discussion of pictorial methods is not, therefore, included in this book, which confines itself to that type of shape description known as orthographic projection.

Shape description by the method of orthographic projection—or an engineering drawing—not only concerns itself with a representation of the object as it appears from several view points but also relates these several views so that a complete description of the object is obtained as it actually is. Moreover, these views are drawn to scale so that not only shape but size as well is described. The theory of this view description and the methods of using such graphic representation for the quantitative determination of values are set forth in this text.

Although the theory of orthographic projection was undoubtedly known as early as 1528 when Albrecht Dürer published his "Menschlicher Proportion," it remained for Gaspard Monge, a draftsman in the employ of the French war department, and later a professor in École Polytechnique, to systematize and establish this theory as an invaluable and fundamental element in the exact method of graphic representation.

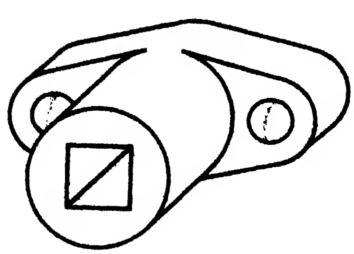


FIGURE 1c
A pseudoperspective drawing: Oblique projection. This type of pictorial representation is used by engineers.

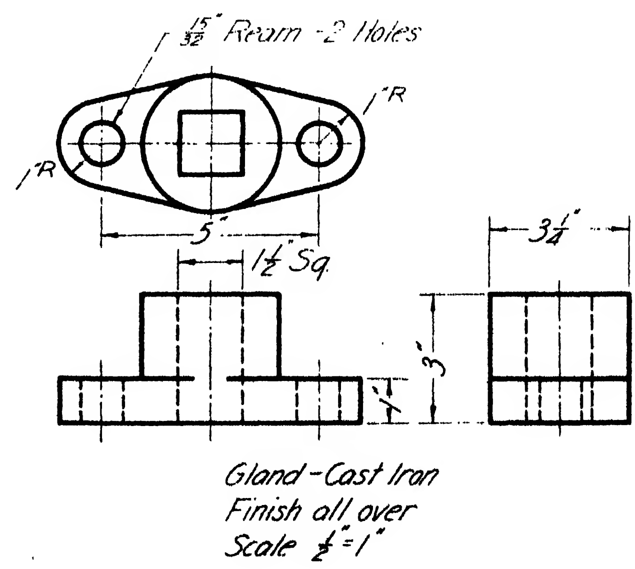


FIGURE 1d
A complete orthographic projection: Working drawing. Size description and shop notes have been added to a three-view shape description.

From France, Monge's system quickly spread to other countries, and through the agency of one of his pupils, Claude Crozet, was introduced at the United States Military Academy in 1816.

From West Point, a military engineering school, the civil engineering schools of the United States adopted the idea.

By various authors, the subject was soon incorporated into textbooks on drawing until now orthographic projection has been accepted and standardized as an integral part of the graphic language. In fact, this system of arrangement of lines and planes and surfaces into views has come to be thought of as the "grammar" of engineering drawing.

6. **The Theory of View Representation.** To represent an object having length, width, and thickness on paper having but two dimensions obviously requires more than one view of the object. These multiple views must be related to one another in such a manner that information on one view may be supplemented by related information from the others.

Such an object, therefore, is considered as being within a box of cubical shape. The object is viewed through the faces of this cubical box (Fig. 2). Upon each of the six faces of the cube, there-

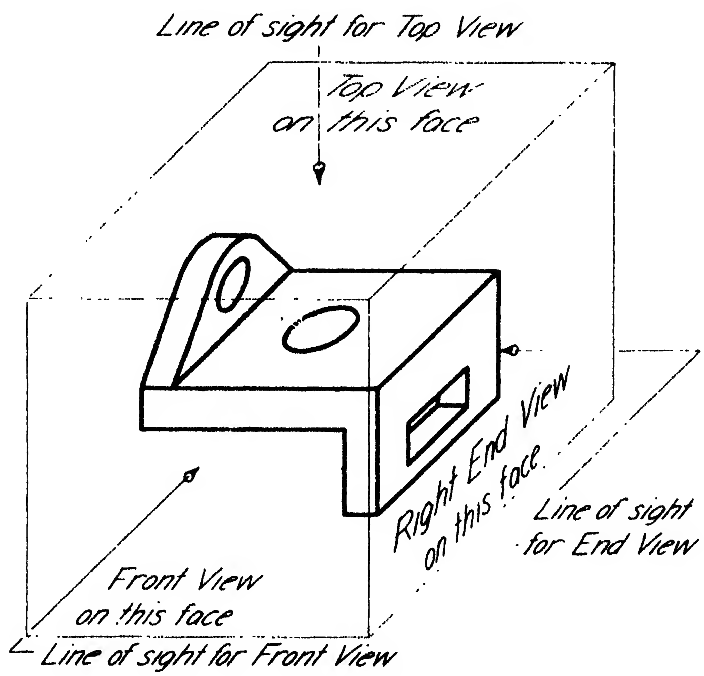


FIGURE 2
Pictorial illustration of an object within a cubical box.

fore, a view of the object may be obtained by projecting (Fig. 3) upon the plane of this face the outlines of the object to be seen when the lines of sight are *perpendicular* to this face. It is evident, then, that by such a scheme of viewing the object six different views of it may be obtained: one view for each of the six faces of the cube.

In order to represent a cubical box of this character upon a flat sheet of drawing paper, and definitely to relate these six views to one another so that information and description may be transferred from one view to others, this box is considered to be opened (Fig. 4) and its six faces placed flat upon the drawing paper. The manner in which this imaginary cube is opened and placed flat—and, therefore, the arrangement and position of the views with respect to one another—has been established by the American Standards Association.

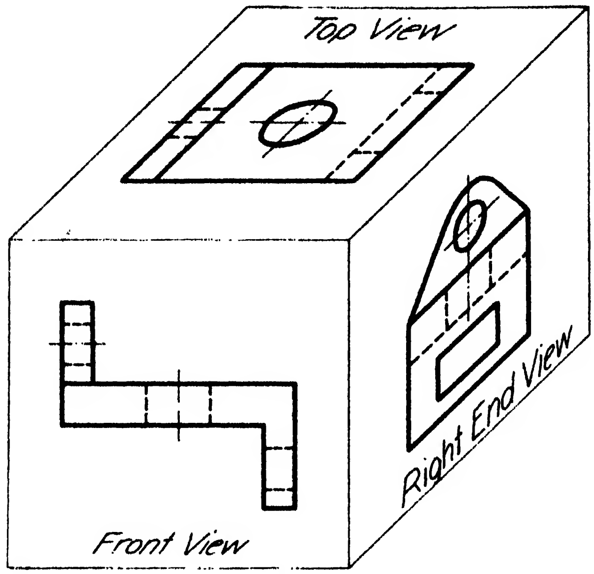


FIGURE 3

Views obtained, shown pictorially, when the object in Fig. 2 is projected on the faces of the box.

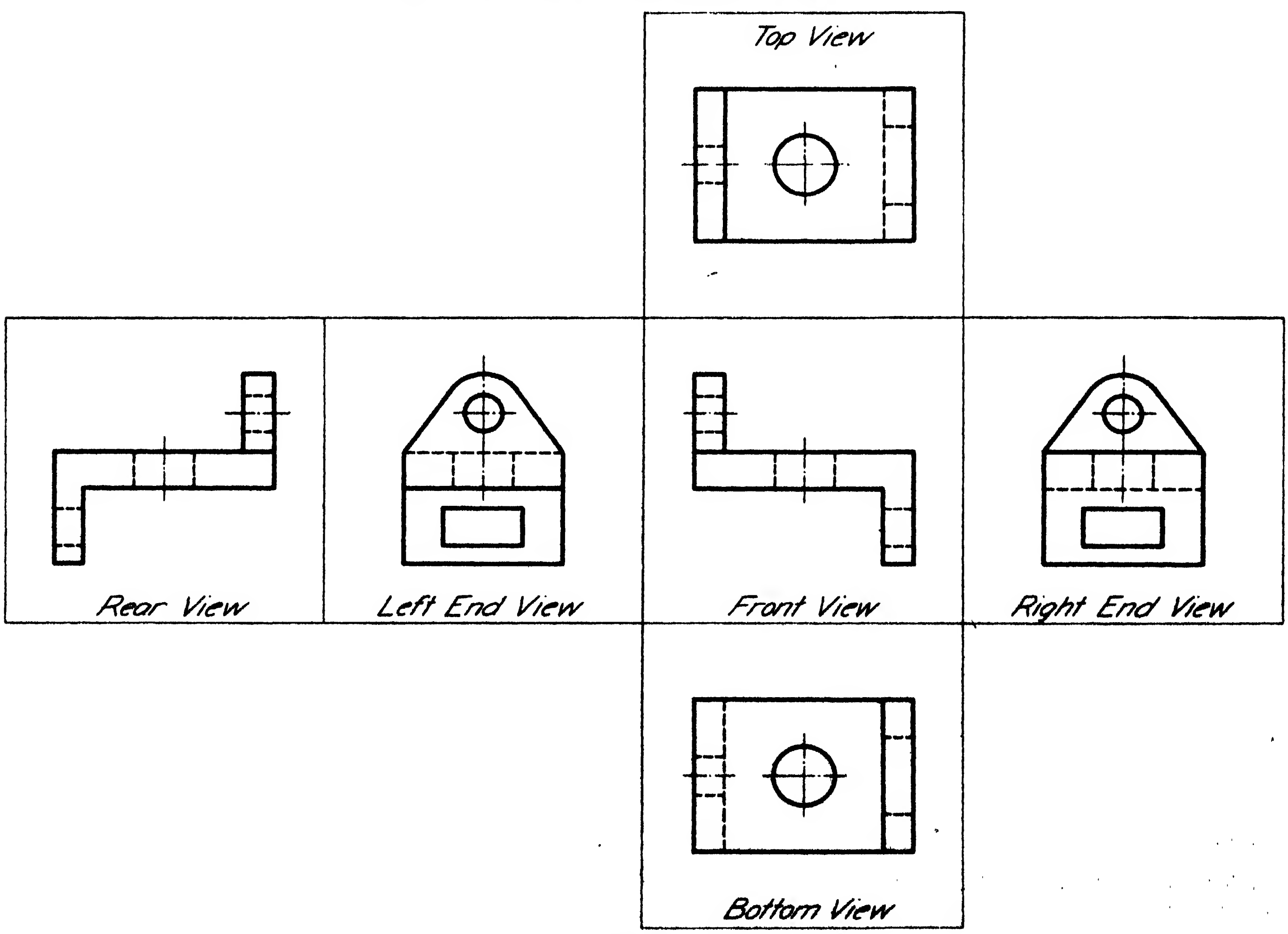


FIGURE 4

The standard arrangement of the six faces of the box, in orthographic projection, with a view of the object on each face.

Wherever drawings are read it is definitely understood that the view of the object seen from above is the top, or plan view; the view seen from in front is the front view, or front elevation; and the views seen from the sides are known as right or left side, or end views, as the case may be; in like manner the bottom view and rear view are designated. Moreover, the position of these views identifies them and their relation to each other is always that shown in Figure 4.

It is to be observed and remembered that the top, front, and bottom views (Fig. 5) lie between *vertical* parallel lines, and that the rear, left end, front, and right end views lie between *horizontal* parallel lines, and that the *front* view is common to both of these sets of views.

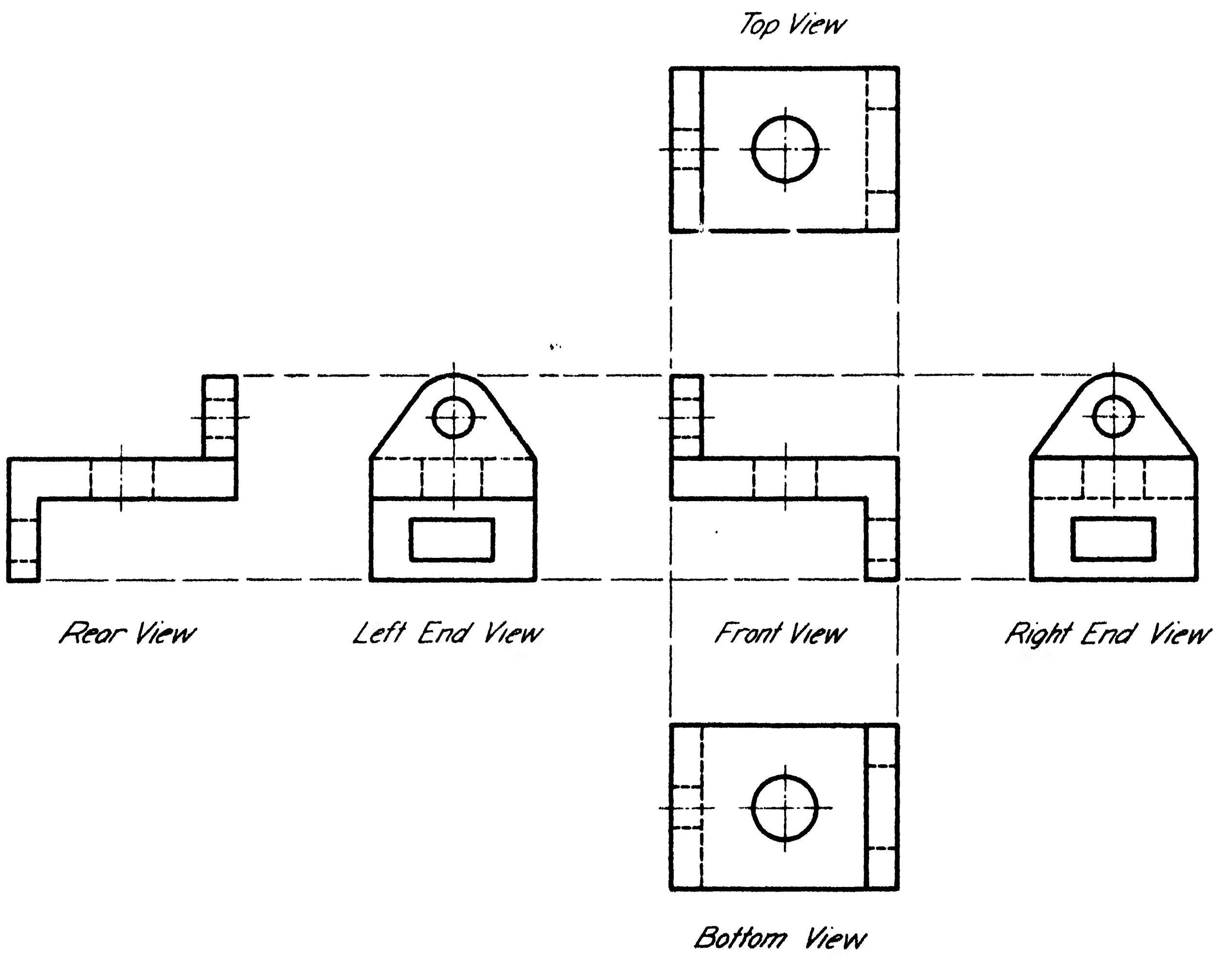


FIGURE 5
The standard relationship and position of the six views.

7. **The Selection and Character of Views.** With the position of the possible views with respect to each other established, the draftsman is now able to "write" in the graphic language grammatically and

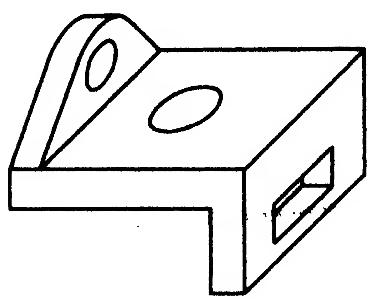


FIGURE 6a

Pictorial view of the object orthographically described in Fig. 6b.

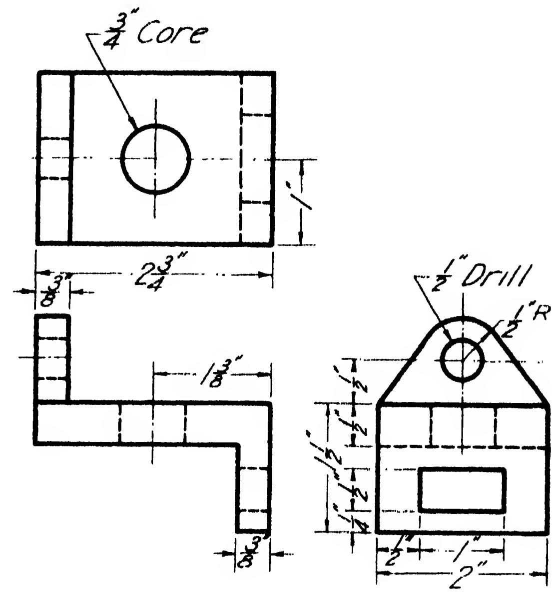


FIGURE 6b

A three view drawing of the object shown in Fig. 6a as a draftsman would make it. Note that reference and construction lines are omitted.

with the assurance that translations of his graphic statements will be uniform and in accordance with the "standard dictionary" of the graphic language. He finds it unnecessary, therefore, to surround these views of the object he is describing with the graphic explanation of how these views were obtained. Accordingly he shows only the views necessary completely to describe the object, and omits from his final representation the construction lines (Fig. 6b) needed for graphical computations and composition of views.

The matter of determining how many and which views completely describe an object requires study and detailed analysis of the object being represented. Such analysis coupled with judgment and experience make the choice and selection of views a relatively simple matter, when it is remembered that three dimensions are almost always involved and that, therefore, in general three views are indicated. Having determined on the character and number of views required, the draftsman now knows where, on his drawing paper, to locate these views and how to relate them to one another so that quantitative determination of size values may be determined.

8. **The Character of the Work** a draftsman has to do and the problems of representing shapes originate in the following ways:

a. As a designer he may create. His problem then is to describe on paper an idea existing only in imagination. This is scientific work of a high order and demands careful graphical expression. In practice, a designer usually describes by drawings enough of his conception to test its workability and then turns his design drawing over to a junior draftsman for the complete description of the whole design and its many parts.

b. As a draftsman he may be required to describe by drawings some part or element of a machine or structure described by a designer on the design layout. He may have to make drawings of some element of a machine or structure described in very general terms on an assembly drawing. He may have to draw a part of a machine or structure already in existence but not shown by drawings, or to draw such a part when only adjacent or mating parts are shown. All such work is generally called *detailing*.

c. He may also be called upon to check drawings for accuracy and correct statement, or to figure from drawings volumes, weights, quantities. Such work requires computation of an exacting nature. And in addition he may, in work of this sort, be required to discover quantities and values by graphical computation.

In all this work, it must be obvious that the ability to describe the shapes of objects, to represent objects so that exact information about shape may be transferred from designer to builder, and the ability to establish in his own mind a clear conception of the shape already described in a drawing are all indispensable qualities.

Whether the draftsman is describing a work existing only in imagination, or a work already outlined and explained by an assembly or design layout, or a work constructed and completed, he must by means of grammatical and correctly arranged orthographic views record the shape he has to describe.

9. **The sequence of steps** taken by a draftsman in drawing the shape description of a part are these:

a. He studies the object and reaches a decision about the number and character of views. That is, he decides that three views, for example, will completely describe the object and selects the top, front, and right end views for this purpose.

b. Having chosen the number and character of views suitable for the description, he now computes the scale to which these are to be drawn. He figures, in other words, from the amount of space available on the drawing sheet used, what fraction of an inch on the drawing paper will represent an inch on the part in order to make the selected views fit the drawing space. Usually an outline sketch is helpful in such estimates.

c. To the scale decided upon, he now locates and draws the center lines, or if center lines are not essential, the working base lines, of all views. These lines in each view are used for the base lines of measurement. Allowance must be made in locating them on the sheet for space for the resulting view and room between views for dimensions to describe size.

d. The working lines for each view having been located, the views may now be built up in unison by working from one view to another by projection and measurement. That view showing the characteristic contour of the part is usually the one to concentrate upon first. This assists in visualizing the shape and makes possible the derivation of other views with a minimum of measurement and a maximum of projection.

e. The final steps in the development of a drawing, as for example the adding of dimensions, or size information, shop notes, and other lettered data, are all elements of engineering drawing and will not be discussed. The steps have been outlined up to and including the point where "drawing by projection" has been introduced, and this is the element of drawing-board geometry now in need of further explanation.

10. **The Principal Views.** While it may be seen from the development of the "imaginary cubical box" that six views (Fig. 4) are possible, it is also evident that six views are seldom necessary to describe an object or a space relationship of three dimensions.

A study of engineering drawing will also reveal the fact that special views have been devised for special purposes: for objects having involved interior or hidden detail, "sectional" views have been devised. In such views the object is cut open along denoted cutting plane lines and viewed as though portions had been removed, thus disclosing the hidden construction. The theory and technic of sectional-view representation is an important part of engineering drawing.

For objects having faces oblique to the faces of the imaginary box, and thus not represented on the faces of the cubical box in true shape and size, special views called auxiliary views have been devised. These views are an important element in drawing-board geometry and are discussed at length in following pages.

Because three-dimensional directions—length, width, and depth—are involved and because views showing these directions are usually adequate for descriptive purposes, the three views which show these are called the principal views, and the three faces of the cubical box upon which these three views are projected are called the principal planes of projection. The horizontal face of the box at the top is called the top, or the *H* plane—*H* for horizontal; the vertical face of the box at the front is called the front plane, or *V* plane—*V* for vertical; and the perpendicular planes at the sides are called end or profile planes with an abbreviation of *P* for profile. The position of an end view at the right or the left of the front view is sufficient to identify whether the end view is right or left. When other considerations do not enter, a right end view is usually shown in preference to a left end view.

11. The positions of these planes upon which the top, front, and end views are projected are indicated by lines (Fig. 7). These lines are in fact *edge views* of the planes; when located, these reference

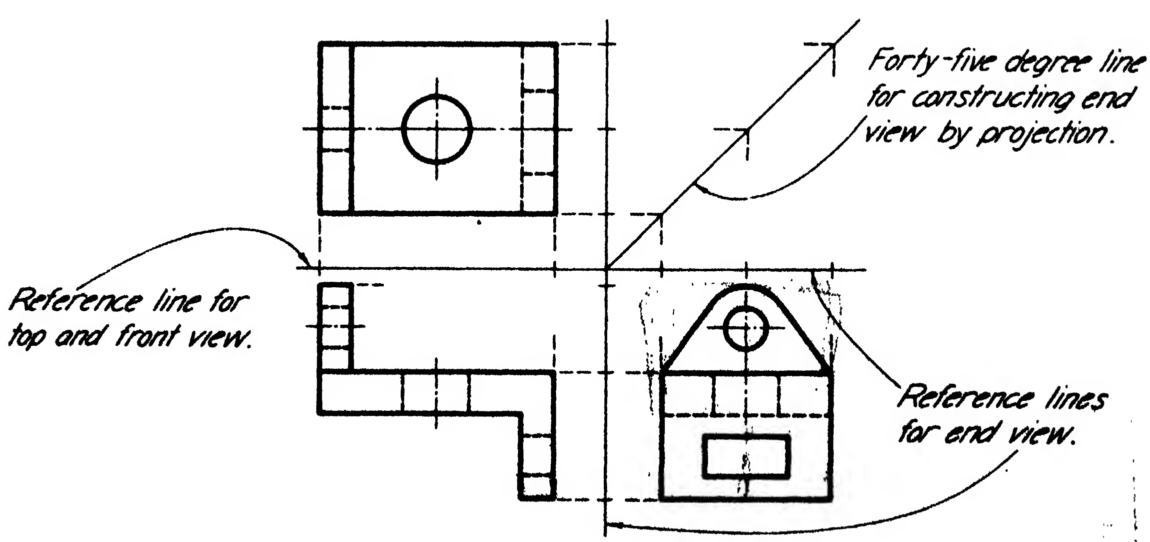


FIGURE 7

A characterstic three-view drawing showing the position of reference lines for top front, and right end plane.

lines denoting the position of the top plane, the front plane, and the end plane become the basic location references for all views.

By using these reference lines—either by actually drawing them in position or imagining them to be there—views may be drawn by projection and measurement with a minimum of tool manipulation, and quantitative facts about the elements of the representation, such as the length of lines, values of angles, and location of intersections, may be ascertained.

Most objects, and therefore many of the three-dimensional relationships involved in their description, are so shaped that a three- or four-view drawing upon the principal planes of projection —*H*, *V*, *P*, or possibly the bottom or rear view plane—is adequate for a complete description. Some one or more of these views may have to be a sectional view, or by some of the conventional devices employed in engineering drawing additional information may have to be added, but so far as the general character and location of the views are involved, a description on the principal planes is usually adequate.

12. When, however, such is not the case, as with an object having faces at angles to the principal planes, or with the solution of a problem wherein values and size information are required concerning a relationship not described in its true shape and size on the principal planes, the device called the *auxiliary* plane is resorted to for description and solution.

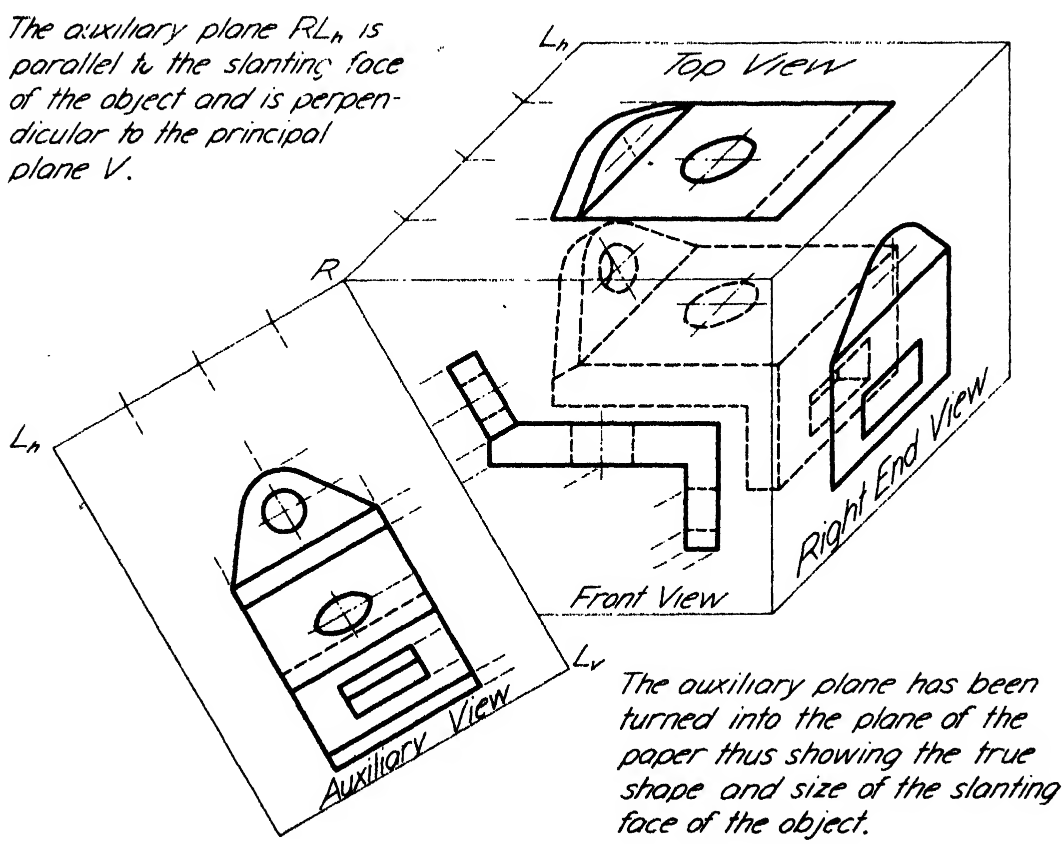


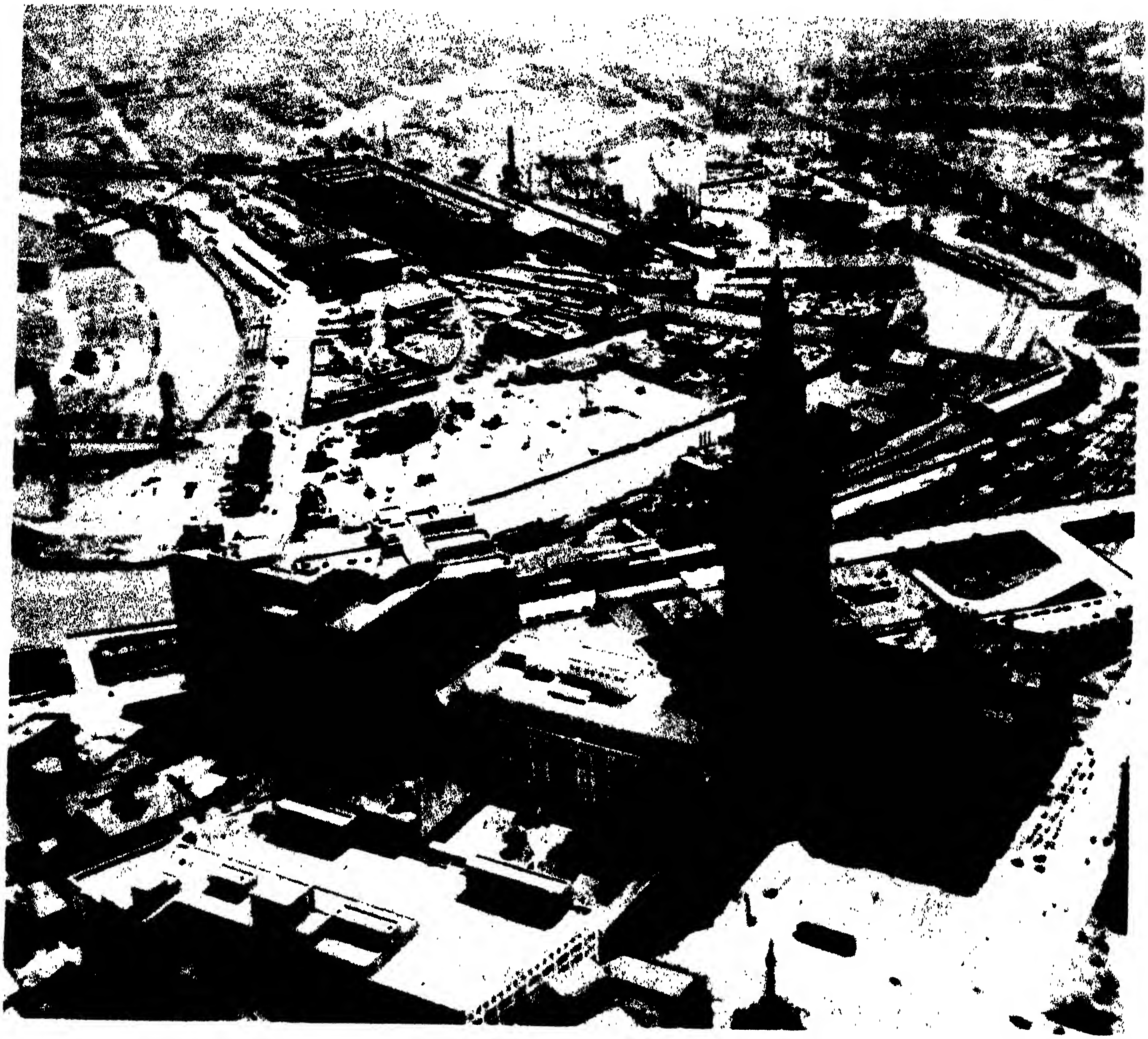
FIGURE 8

A "projection box" showing the position and relationship of an auxiliary plane.

An auxiliary plane has one outstanding characteristic: whereas the principal planes—*H*, *V*, and *P*—are mutually perpendicular as may be readily comprehended by reference to the cubical box (Fig. 2) from which they are derived, the auxiliary plane is perpendicular to *one* and *one only* of the principal planes and inclines at an angle designed to make it parallel to the space situation it has been chosen to describe.

In all other respects, auxiliary planes are like principal planes: projections and views are obtained upon them by viewing the object or space relationship to be described in a perpendicular direction to the plane. They are represented by reference lines which are in reality edge views of such planes, and their position with reference to the principal planes is determined by the location of such reference line.

They make it possible, in fact, for the draftsman to describe in true shape, size, and relation any object or space relationship not so described by representation on the principal planes of projection.



An aviator's view of engineering. Every phase of engineering is illustrated: design, construction, operation, and maintenance in all branches of the profession are shown in the picture. Countless drawings and graphical solutions are required to make such a development possible.

CHAPTER II

DRAWING-BOARD GEOMETRY

13. **Definition.** Drawing-board geometry is a form of graphical computation by means of which quantitative values are obtained for relationships described by an orthographic drawing. There are many relationships described by orthographic projection which need more views of a fundamental nature, or which need to be projected on planes other than the principal planes, in order to show these relationships in their true shape, size, and value.

The principles of projection utilized for this purpose, and the drafting manipulations required to disclose this additional information, are called "drawing-board geometry" because the problems originate on the drawing board, or may be stated conveniently by drawings, and are solved by drawing-board methods employing geometrical principles.

14. **The General Methods of Solutions.** The problems conveniently and expeditiously solved by drawing-board methods involve the angular relationship of lines to other lines, and to planes; the true length and slope of lines; the true shape and size of plane areas; and the angular relationship of planes.

Such problems are commonly solved by drawing-board geometry: (1) by obtaining a fundamental view of the relationship on a suitably chosen new plane of projection called an auxiliary plane, (2) by changing the position of the object so that a fundamental view may be obtained on one of the principal planes of projection, or (3) by a combination of these two methods.

The first-named method has been called the "direct" method; it depends upon viewing the relationship to be solved from additional positions represented by auxiliary planes. In other words, the *position of the observer is changed* from that used for the principal planes. (See Fig. 9*a*.)

In the second method no new plane of projection is required but the *position of the object* is changed so that revolved views upon the principal planes are adequate for the solution. See Fig. 9*b*.

The second method is a part of the method used in descriptive geometry, the subject which was systematized by Gaspard Monge in 1795 and is in common use in engineering schools everywhere. The study of descriptive geometry has much to commend it: it develops the power of analytical thinking and of visualization and provides a graphical method for the solution of all drawing-board problems.

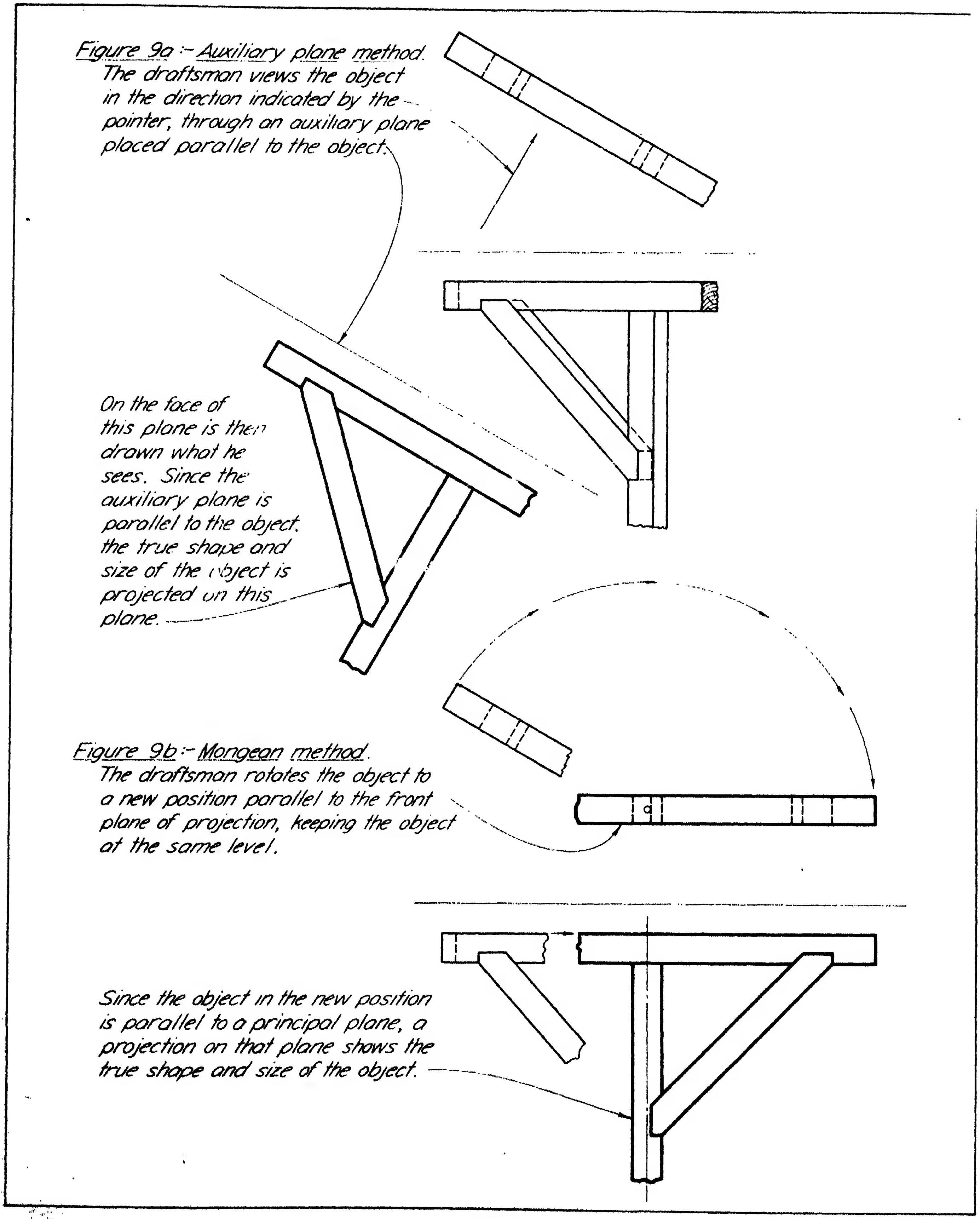


Figure 9

Illustrating the "change of viewpoint" and the "change of position" methods.

The first method also is a part of the method used in descriptive geometry simplified so that most of the major problems in space relationships which are met in drafting and design work can be solved.

The important differences in the two methods lie in the use of *oblique* planes: whereas the method of descriptive geometry depends upon revolution and oblique planes for the solution of problems, the method of drawing-board geometry utilizes the method of revolution to a limited extent, eliminates oblique planes as a vehicle for carrying out solutions, and depends almost entirely upon auxiliary views.

15. **The Scientific Method.** One of the outstanding reasons why engineers are successful in fields of activity other than engineering is that they are trained during their college course in the scientific method. The solution of problems by this method is followed in drawing-board geometry and involves the following steps:

A. **The collection of facts relating to the problem.** In engineering work this involves a survey of all the known factors which may influence the solution. The nature of these factors and the method of securing information about them will depend upon the character of the undertaking. The design of a machine, the construction of a bridge, the erection of a building, the improvement of a transportation system, the financing of municipal improvements, and the building of a power plant are all illustrations embodying the fundamental necessity of discovering in advance all the engineering and economic elements affecting the proposed undertaking. The problem must be solved successfully not only from the standpoint of design and construction but also from the economic standpoint of whether the proposed undertaking is financially sound.

B. **The recording of data and stating the problem.** When the facts are collected, the necessity for making a record of them in such logical manner and clear form that they may be preserved, easily reviewed, studied, and analyzed is obvious. The use of graphics in this step of the scientific method is indispensable. Maps, profiles, charts, graphs, drawings, and tabulated statistics play an important part in presenting information about the problem and also provide ready means for a statement about its many details in a form suitable for solution of the problems involved.

C. **The analysis of the problem and the possible methods of solving it.** In addition to having the problem defined and knowing all the factors influencing its solution, technical knowledge and scientific laws must be understood before a complete analysis of the problem can be made and a decision reached concerning the best method of solution consistent with technological and economic considerations. As analysis of the problem continues in the light of all known factors, certain methods of solution become obviously impractical or uneconomical. By elimination and study the problem is reduced to its elements and a method for solution is discovered.

D. **The solution of the problem.** After it has been found by study and analysis that the problem can be solved in a practical way, and that economically it is *worth solving*, the solution becomes the final step in the scientific method. The first element in the solution may be a report on the project outlined in detailed written form and illustrated by charts, diagrams, tabulations, photographs and pictorial sketches, drawings, and (as Kipling once stated in this connection) "acres and acres of computations." With the approval of the project secured, its accomplishment

financed and authorized, an extensive and cooperative organization must now be established to design and build and assemble all the many elements of the project into a component and functioning whole.

Thus, by breaking down the attack upon an engineering problem into the essential and sequential elements of procedure, and by treating each of these elements as a problem by itself to be analyzed and solved before moving on to the next step, a method of work is established by means of which fundamentals are evaluated and a solution obtained by direct and purposeful procedure.

16. **The Statement of the Problem.** As in practically all problems in every kind of work, the collection of essential data bearing on the solution of the problem, and especially the statement of the problem, whether in the form of a mathematical equation, a written specification, or a drawing or graphical statement, is the preliminary step of fundamental importance.

In the work to which this book limits itself, data for problems are presented either by a written specification setting forth the known facts, or in the form of a drawing—either a complete drawing or a drawing in outline form from which for simplification non-essentials have been stripped—or by a combination of written statement and drawing. The draftsman, at the outset of his attack on a problem, must discover the essential factors bearing on the solution of the problem and set these down on paper in graphical form for study, analysis, and eventual solution for the unknown but discoverable values of which he is in search. Thus it will soon be discerned that (1) the solution of problems in drawing-board geometry is admirable and direct training in the scientific method; (2) the graphical statement of known facts about a problem is an invaluable aid to a complete appreciation and understanding of the situation; and (3) many kinds of problems in engineering work may be studied and analyzed advantageously by this method and, indeed, solved if the precision of the computations are within the limits of drawing-board work.

As an example of this, let, the following problem be chosen as an illustration.

A. **The collection of facts relating to the problem.**

An object floating on the water is observed from a lighthouse 100 feet above the water at an angle of depression of 30°. At ten o'clock, when first observed, the object is N30°E from the lighthouse. One hour later the same object is observed through an angle of depression of 15° and N60°W from the lighthouse. At what rate of speed is the object drifting, and upon what course?

B. **The recording of data and stating the problem.**

Sketching is an invaluable aid in presenting the known facts and for discovering what unknown facts are to be found.

The distance from the base of the lighthouse to the object must be found in order to locate its position when first observed. The distance from the base of the lighthouse to the object must also be found in order to locate its position one hour later. These two positions having been located, the direction and rate of drift may now be determined.

Merely by reading the above statement, the need for a "picture" of this problem situation makes itself felt. Even in analytical solutions, diagrams are needed to fix these relationships in

mind. If the problem were presented to an engineer for solution, his very first reaction would be to express all the known factors in a graphic form for study and analysis. This form of expression is an invaluable aid in clarifying a problem and making it easy to grasp its essential factors.

C. **The analysis of the problem and the possible ways of solving it.** Since the graphic diagram (Fig. 10) indicates a triangular relationship, and further since the given known factors are stated in

The solution of the problem. By trigonometry.

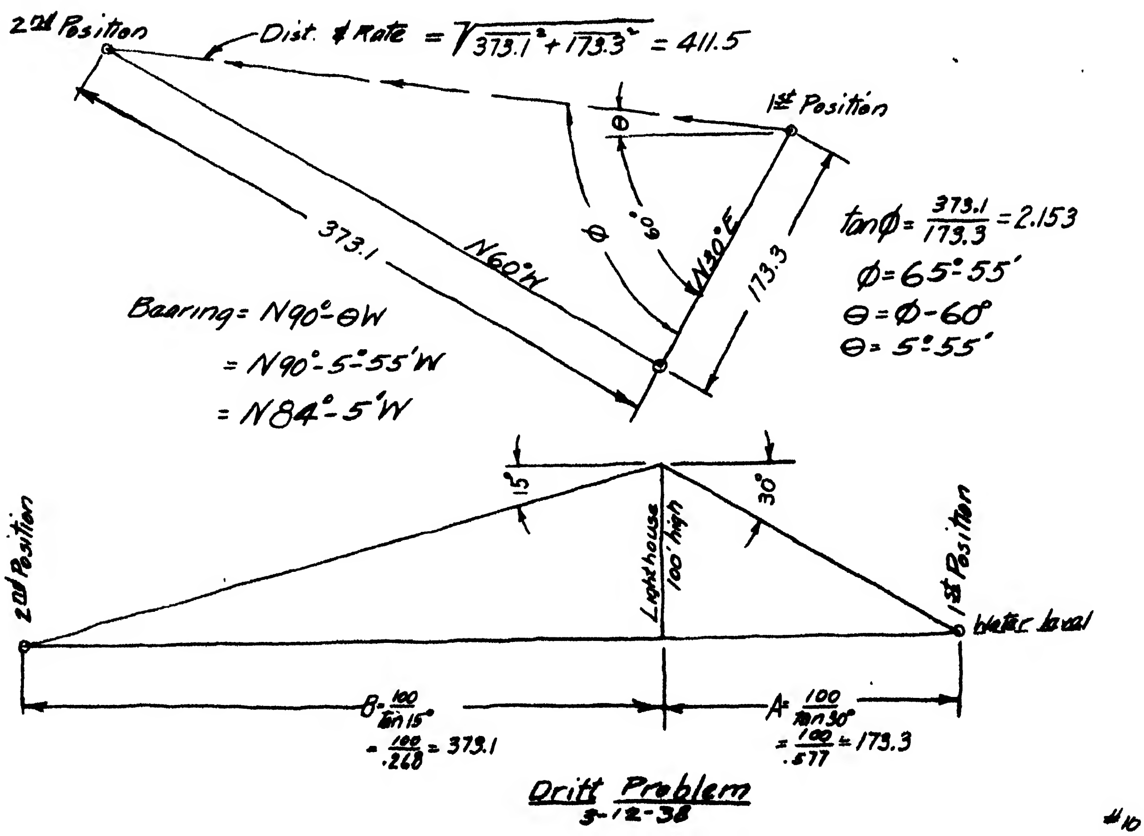


FIGURE 10

A freehand sketch and computations for the solution by trigonometry.

angular values, the method of trigonometry naturally suggests itself as a means of solving the problem. By this method the length of A may be found by using the right triangle formed by the distance A as a base, the height of the lighthouse H as an altitude, and the angle of depression of 30° $\cdot \frac{H}{A} =$ tangent of 30°; A therefore will equal the tangent of 30° divided into H. In like manner B may be found. Thus the two positions one hour apart as to time of observation may be plotted and the length and direction of this distance computed.

D. **The solution of the problem. By graphical measurement.**

However, since the graphical presentation of the problem suggests a scale drawing which not only will describe the problem but also will enable the values required to be measured directly, this method should be considered. The values of A and of B may be determined by projection, and the direction and speed may be measured; except for comparative precision the graphic method affords some advantages.

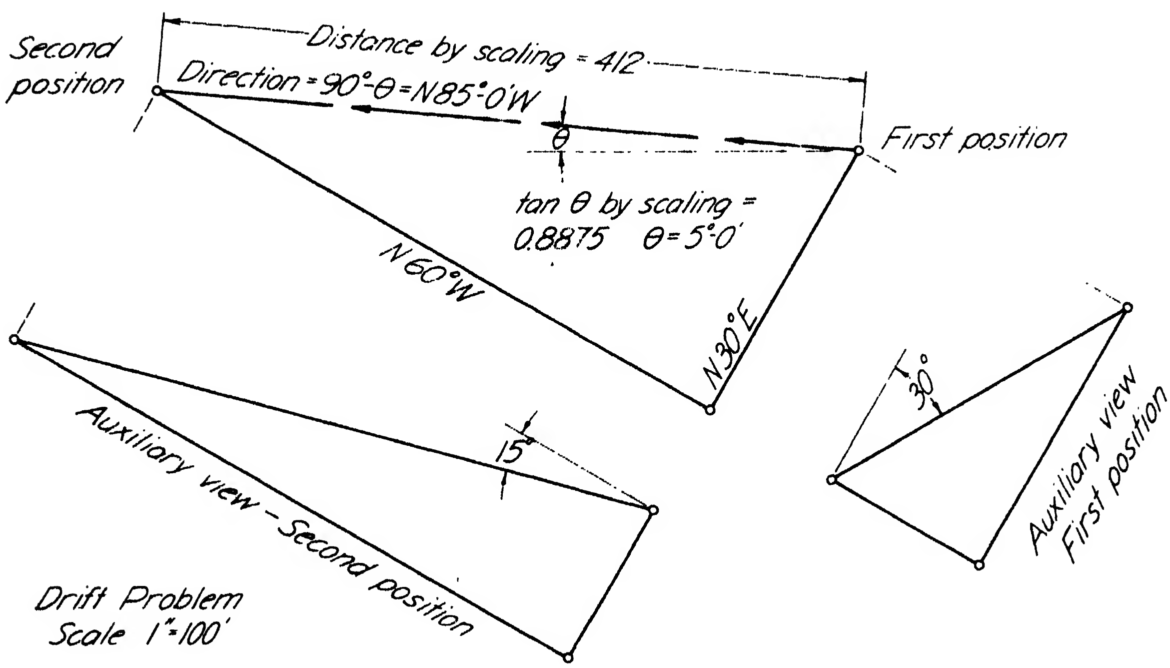


FIGURE 11

A drawing from which the required data may be measured.

17. The Auxiliary Plane Method. It will be noted that the strategy in the attack on this problem depended on two prime factors: (1) knowing what to do by analysis and (2) locating on the drawing auxiliary planes upon which could be drawn the relationships involved so that these appear in measurable position.

Thus, the essential principle of solution lies in locating auxiliary planes upon which may be drawn points, lines, planes, surfaces, and their relations to each other in such a manner that their true shape, slope, size, and relationship are disclosed in measurable form on the drawing. Such, then, is the object and method of drawing board geometry.

CHAPTER III

FUNDAMENTAL VIEWS

18. **The Elements of Representation.** Analysis of engineering structures reveals that they are made of shapes more or less familiar such as cylinders, prisms, and other geometrical forms. To describe these forms, either as single units or in combination with other units, requires their representation by two or more views. These views in turn are made up of graphic symbols such as points, lines, and planes, which become the elements of representation. To represent these elements by views, to recognize from a graphic symbol what kind of an element (Fig. 12) is being represented, and to be able to

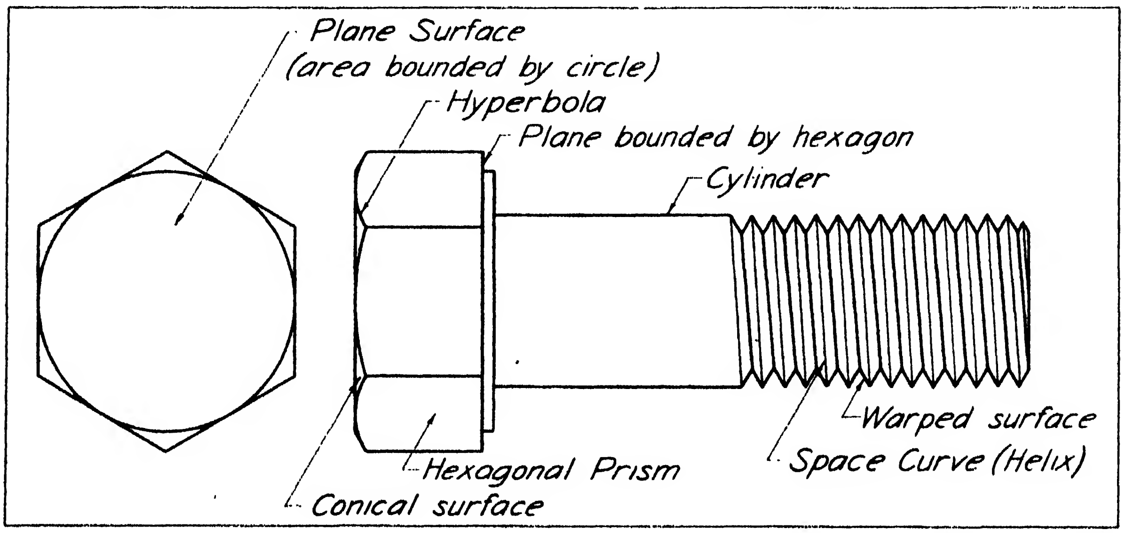


FIGURE 12

ascertain the relationship of this element to others and to the whole structure constitute the fundamental requirements of drawing-board geometry.

19. **Definitions.** For the purposes of this book, a point may be considered a location; a line may be considered as an edge, the limit of an area, a contour element, or a direction; a plane may be considered as an area either bounded by lines and therefore having shape, or an area of unlimited extent defining merely direction and slope. From these definitions it should be evident that a point is usually associated with a line—it may be the beginning or end of a line, or any location on the line; that a plane is an area defined by lines—the area may be bounded by three or more straight lines or one curved line, or it may have its position fixed by containing two lines determining direction and slope. Therefore, it may be seen that the fundamental element in graphic representation is the line.

Furthermore, a study of the representation of the line in all its possible positions in space will make the important fact evident that all problems in drawing-board geometry having to do with points, lines, planes, and curved surfaces may be solved by locating the critical *line* of the problem and finding the fundamental view of this element of the representation.

20. **Necessary Views.** One of the fundamentals of good drawing in representing an object is to describe the object completely but to show only as many views as are needed for this purpose and no more.

The same principle applies to drawing-board geometry. Obviously two views are always required to describe a three-dimensional relationship; usually a third view is necessary. In drafting, the commonly used views are top and front, or front and end, and if three views are needed the combination is usually top, front, and right end.

In drawing-board geometry the top and front views are the usual views chosen to record the data, and when a third view is needed this is generally the right end view.

When such views do not suffice, and other views are required, the auxiliary view is resorted to as the means for describing the relationship. Since it is possible to obtain an indefinite number of auxiliary views, the principle of engineering drawing referred to should be the guide in choosing the views to show, namely, select only those views which will completely describe the relationship and present in measurable form the solution required, and draw no others. The best drawing is that which gives all the facts with the least number of lines.

21. **Notation.** On working drawings, reference letters are seldom needed. The location of section planes to identify sections is often denoted by letters, and there are other occasional uses for identification of this type. On drawings which are reduced to simplest terms and consist, therefore, mainly of points, lines, planes, etc., identification and reference letters are helpful in relating the several projections or views of these elements of description. Capital letters such as *A, MN, MNOP* are used in text matter to indicate the location in space of a point, a line, or a bounded plane area. On the drawing *the views* of these are marked with lower-case letters, as *a, mn, mnop,* and the *position of the view* identifies it as top, front, end, or auxiliary view. Sometimes numerals are used in place of letters.

The character of lines used in representing points, lines, planes, etc., is identical with engineering-drawing practice. Projection lines, or those lines joining with several views of a point, are light dashed lines, but since, for simplicity, construction lines are used as sparingly as possible, these projection lines are not drawn longer than sufficient to indicate their position. For accuracy, reference lines and construction lines are drawn with light lines; given graphical data and found graphical data are somewhat heavier and accented for emphasis.

22. **Projection Planes.** The principal planes upon which views are drawn are six in number for normal views, and of an infinite number for auxiliary views. The position and relation to each other of these planes are indicated on the drawing by reference lines—marked *RL* with a subscript *h, v, p* to indicate in which principal plane it lies—which are in reality edge views of these planes. Figure 13 shows the reference lines for the three normal views ordinarily used—top, front, right end—and identifies the reference line for each.

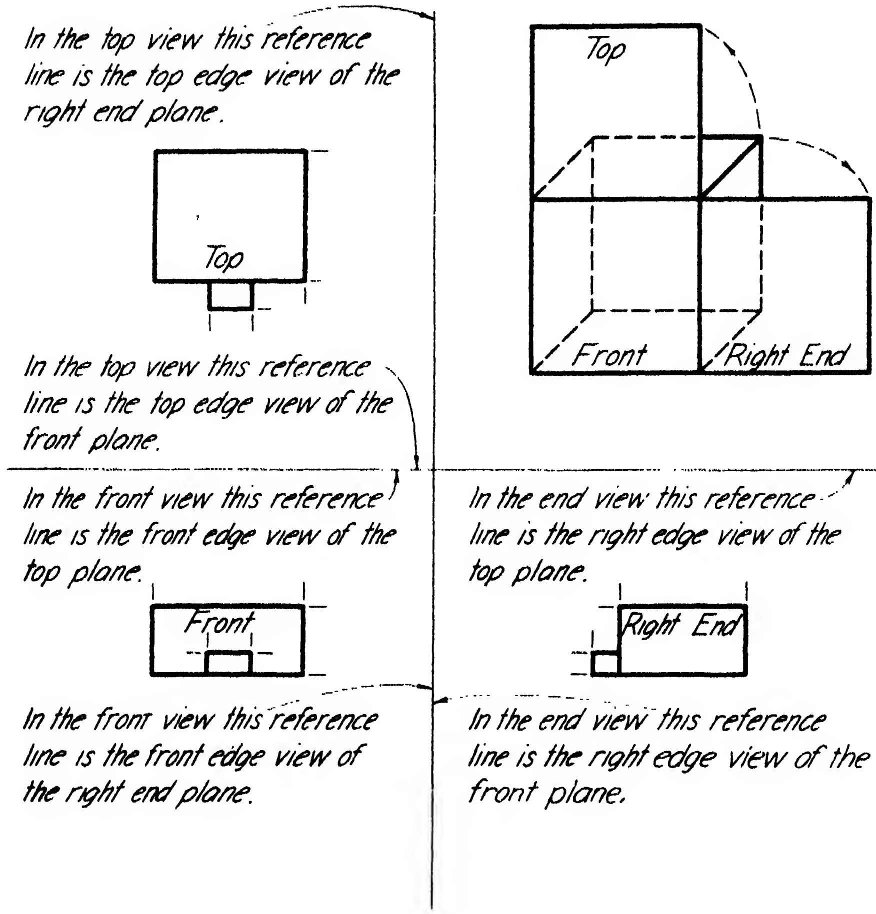
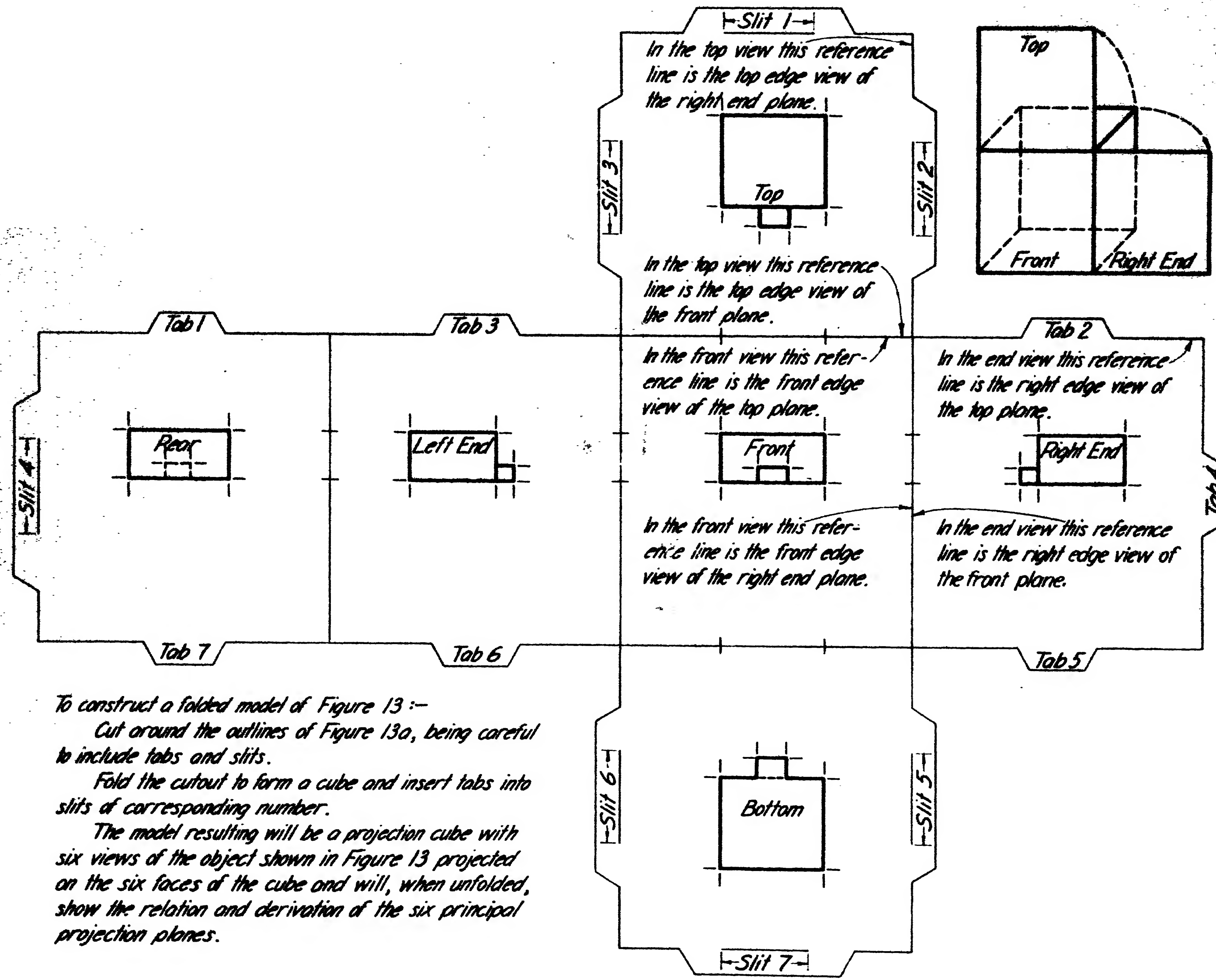


FIGURE 13

The reference lines for top, front, and right end view.



To construct a folded model of Figure 13:—

Cut around the outlines of Figure 13a, being careful to include tabs and slits.

Fold the cutout to form a cube and insert tabs into slits of corresponding number.

The model resulting will be a projection cube with six views of the object shown in Figure 13 projected on the six faces of the cube and will, when unfolded, show the relation and derivation of the six principal projection planes.

Figure 13a

Remove this page from the book. Cut and fold as directed, and a model of Fig. 13 will be formed.

Figure 14 shows the reference line positions for a top, front, left end, and an auxiliary view. While auxiliary planes are of indefinite number, only those are used which serve the requirements of the problem. The position of these planes is, therefore, fixed by two requirements: (1) they must be perpendicular to some one principal plane; and (2) they must be located as to slope with specific reference to the problem they are to serve.

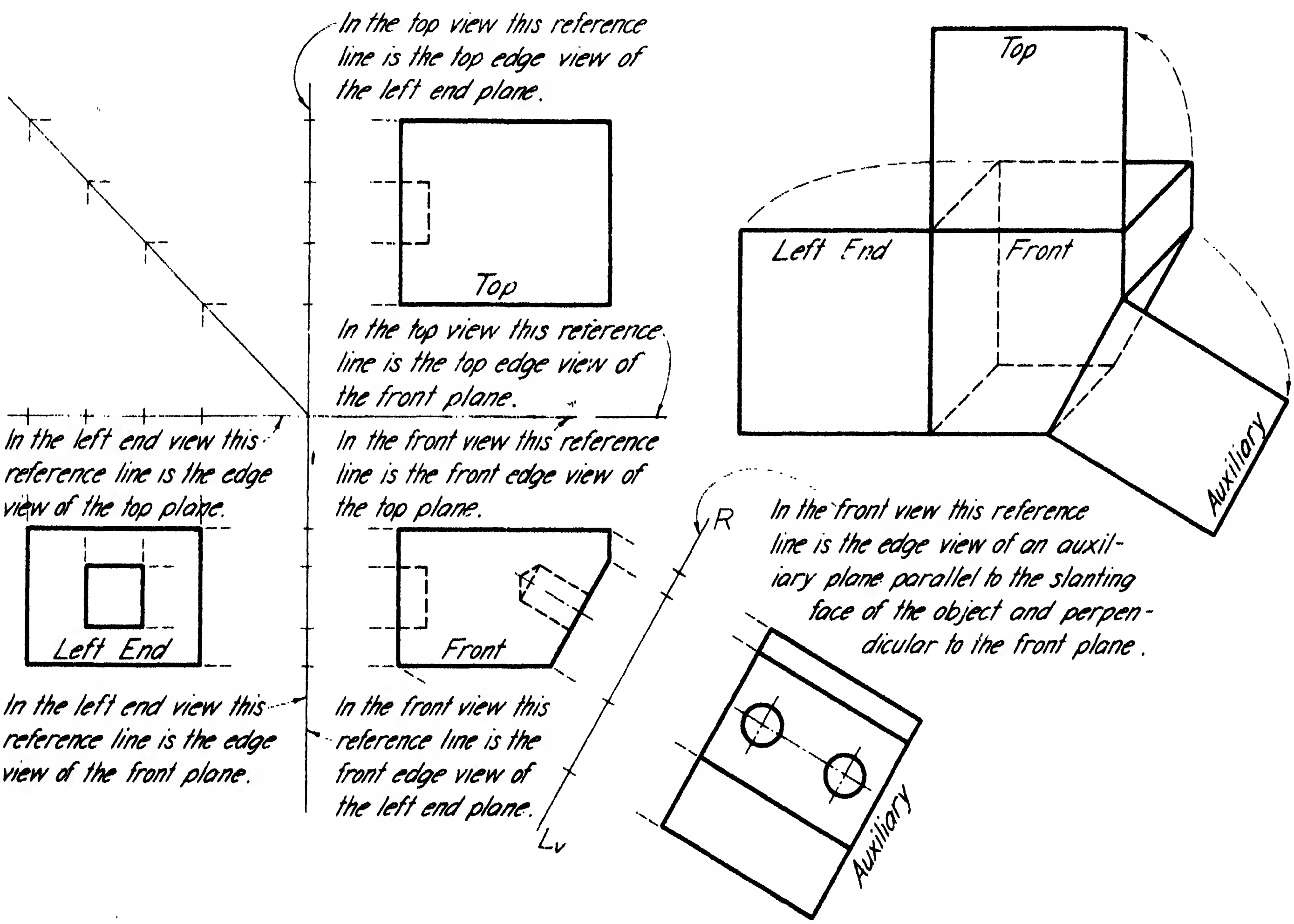


FIGURE 14

The reference lines for top, front, left end, and an auxiliary view.

In text matter and problem statements the planes of projection are often referred to by letter, as *H* for the horizontal or top plane; *V* for the vertical or front plane; *P* for profile or end planes with the position of the view indicating whether it is right or left; and *A* for auxiliary planes.

23. To solve any problem by the graphical methods of drawing-board geometry, lines of reference (RL) must be established in order to relate the elements of the problem (such as points, lines, and planes) to the planes of projection as a base for measurement and for transferring distances from one view to another. When these reference lines are drawn, the distance of points, lines, and planes from H, V, and P (top, front, and profile planes of reference) become established.

In Fig. 15*a*, for example, a front and a right end view of an object are given. The problem is to make a better shape description of this object by adding a top, or plan, view by projection. The first step, then, in this graphical solution is to locate the necessary reference and projection lines.

In Fig. 15*b* is shown these required reference lines. the method of locating them, and the derivation of the required top, or plan, view by their use.

In Fig. 15*c*, three views (a plan, a front elevation, and a right end view) are given showing a complete description of the object. The draftsman's problem is to locate the necessary reference lines on the drawing so that certain changes in the object may be represented and drawn by projection.

In Fig. 15*d*, these required reference lines are shown, the method of locating them is indicated, and the changes in the shape of the object are represented and drawn by projection. It is to be noted that the notch of given width and depth which was to cut through the top of the object introduces new lines in the drawing, and that the location and position of these new lines are found by measurement and projection.

When a draftsman lays out a problem, the position of the reference lines is fixed only by space requirements of the resulting drawing. In such a case it should be kept in mind in planning the drawing that it is important to have views reasonably close together for ease in reading the drawing, that the views should present a well-arranged and balanced appearance on the paper, and that sufficient space should be provided around the views for notes and size description which will be added later.

To construct a folding model of Figure 14:—

Cut around the outlines of Figure 14a, being careful to include the tabs and slits.

Fold the cutout to form a projection box as shown pictorially in Figure 14 and insert tabs into slits of corresponding number.

The model resulting will be a projection box with an auxiliary plane as illustrated in Figure 14.

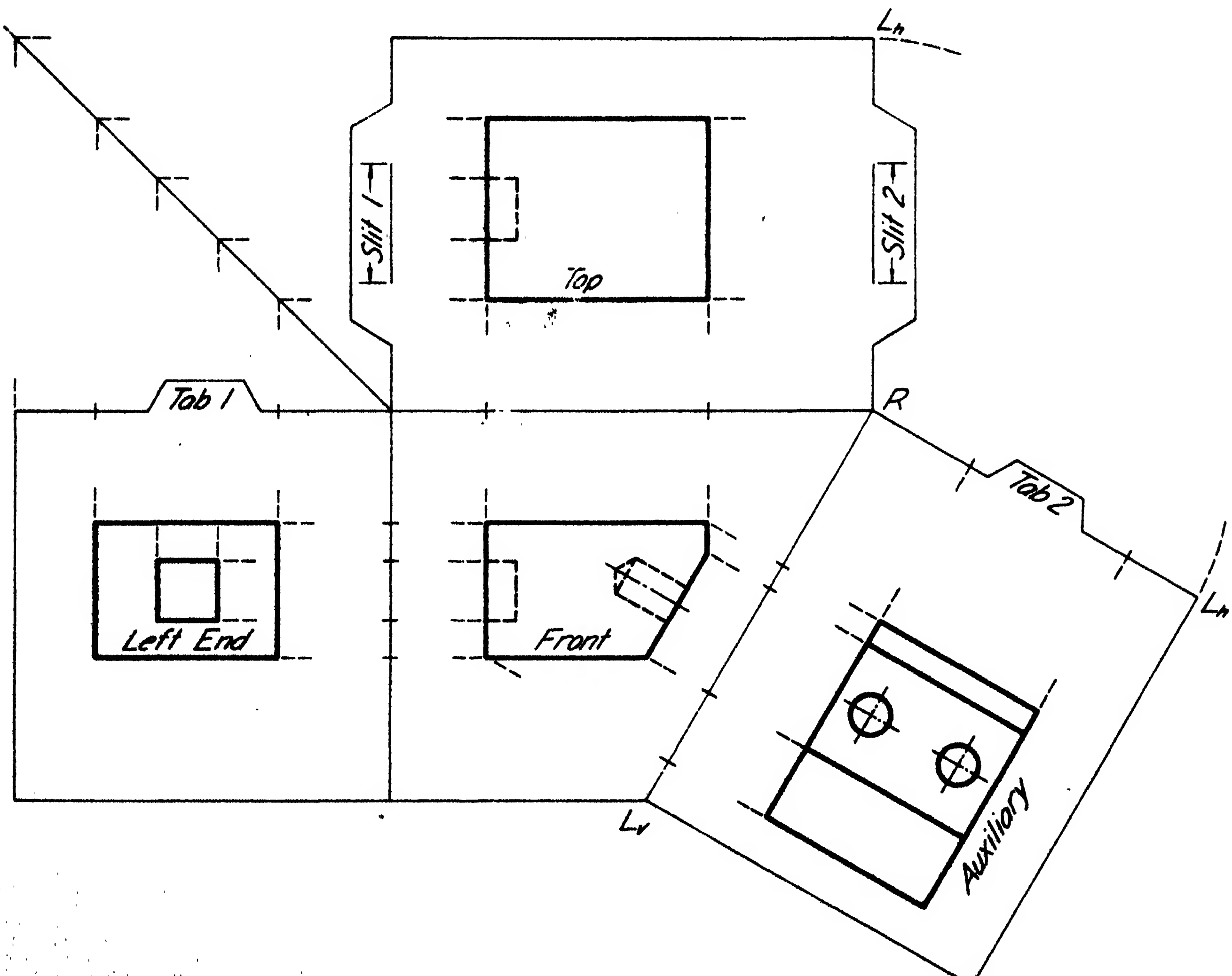


Figure 14a

Remove this page from the book. Cut and fold as directed, and a model of Fig. 14 will be formed.

To locate reference lines for a two view drawings:—

Draw the first reference line (1) in a convenient position between the two given views.

Draw the second reference line (2) a suitable distance from the given view: in this case the front elevation. Next draw the 45° projection line through the intersection of reference lines (1) and (2).

The drawing is now provided with reference and projection lines for drawing a third view by projection.

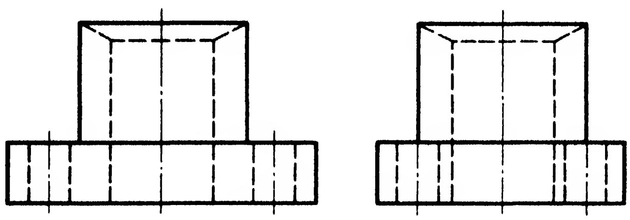


Figure 15a:—A two view drawing without Reference Lines.

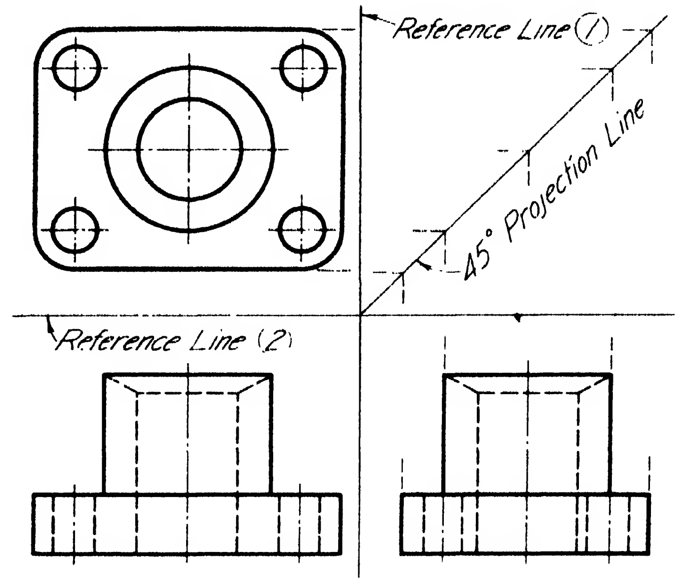


Figure 15b:—The same two view drawing with Reference Lines located and a third view derived by projection.

To locate reference lines for a three view drawing:—Draw the 45° projection line through the point of intersection of a horizontal line from the top view and a vertical projection line from the end view.

Draw the end view reference line (1) in a suitable position between the front and end views.

Draw the top and front view reference line (2) through the point where the 45° projection line and reference line (1) intersect.

The drawing is now provided with reference and projection lines properly related to the three given views.

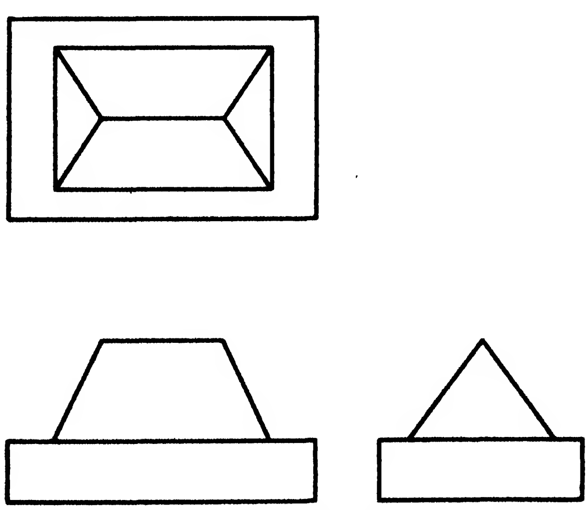


Figure 15c:—A three view drawing without Reference Lines.

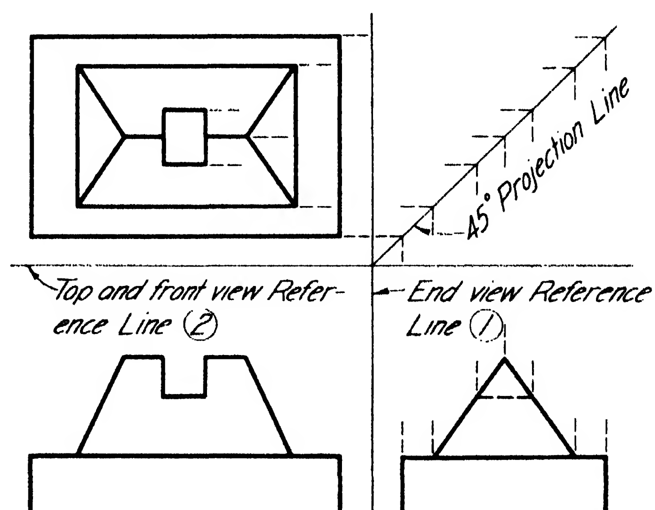


Figure 15d:—The same three view drawing with Reference Lines located and changes made in the object derived by projection and measurement.

LOCATION OF REFERENCE LINES

FIGURE 15

The methods used for locating reference lines in orthographic projection.

24. **Possible Line Positions.** A line may be in one of seven different basic positions in relation to the principal planes of reference. These positions are graphically described in Figs. 16 on the following pages.

In Fig. 16*a*, three positions of the line are shown when the line is perpendicular to the principal planes. Certain obvious relationships are to be observed in studying these graphic descriptions.

Top Row. Line *AB* is one edge of a block shown pictorially at the left. This edge *AB* is perpendicular to the top, or horizontal, plane of projection. Therefore, in the three-view drawing of the block, the line *AB* is represented by a point (*ab*) in the top view. Since *AB* is perpendicular to the top plane, *AB* must of necessity be parallel to both the front and profile plane. Consequently the front view *ab* and the end view *ab* are equal in length and are equal in length to the line itself. The edge *AB* alone is shown as an orthographic projection in the figure in the right-hand figure of the top row. From such a three-view drawing, the relation of the line to the three principal planes and its distance from them may be observed. From the dimensions on this drawing, it is to be noted that *AB* is a *v* distance *behind* the *V* plane; that *A* is an *h* distance *below* the top plane; that *A* and *B* are a *p* distance to the *left* of the end, or profile plane; and that the length of *AB* may be measured by the distance *TL*. Furthermore, it is to be well noted that these dimensions all appear in two views.

Middle Row. The pictorial drawing at the left shows the position of the block arranged so that *AB* is now perpendicular to the front plane. In the three-view drawing of the block, therefore, the edge *AB* is represented in the front view by a point (*ab*). Since *AB* is perpendicular to the front plane, *AB* will be parallel to both the top and end planes. Therefore, the top view and right end view *ab* are equal in length and are equal in length to the line itself. The edge *AB* as a line alone is shown as a three-view drawing at the right. From the dimensions on this drawing, the distance the point *A* is from and its position with respect to each of the three reference planes may be read. The position of *B* and its distance from these planes may be found by similar measurements. The true length of *AB* is also shown.

Bottom Row. The position of the block in this description is adjusted to make *AB* perpendicular to the end, or profile, plane. In the three-view drawing of the block, the edge *AB* appears on the right end plane as a point (*ab*). Since *AB* is perpendicular to the end plane, the line *AB* will be parallel to both the top and the front plane of projection. Therefore, these two views of *AB* are equal in length, are equal in length to the edge *AB* itself, and are parallel to the horizontal reference line between the *H* and *V* planes of projection. In the right-hand drawing of this series, the edge *AB* is represented as a line, or one of the geometric elements of shape description. From the dimensions on this three-view description, the position and relation of *A* and *B* to each of the principal planes of projection may be seen. The true length of the line is also shown.

Suggestion: With dividers check the distances from *H* and *V* in the end view in the drawings, compare these with the same distances in top and front views, and note particularly how these distances are transferred and how views are derived by projection.

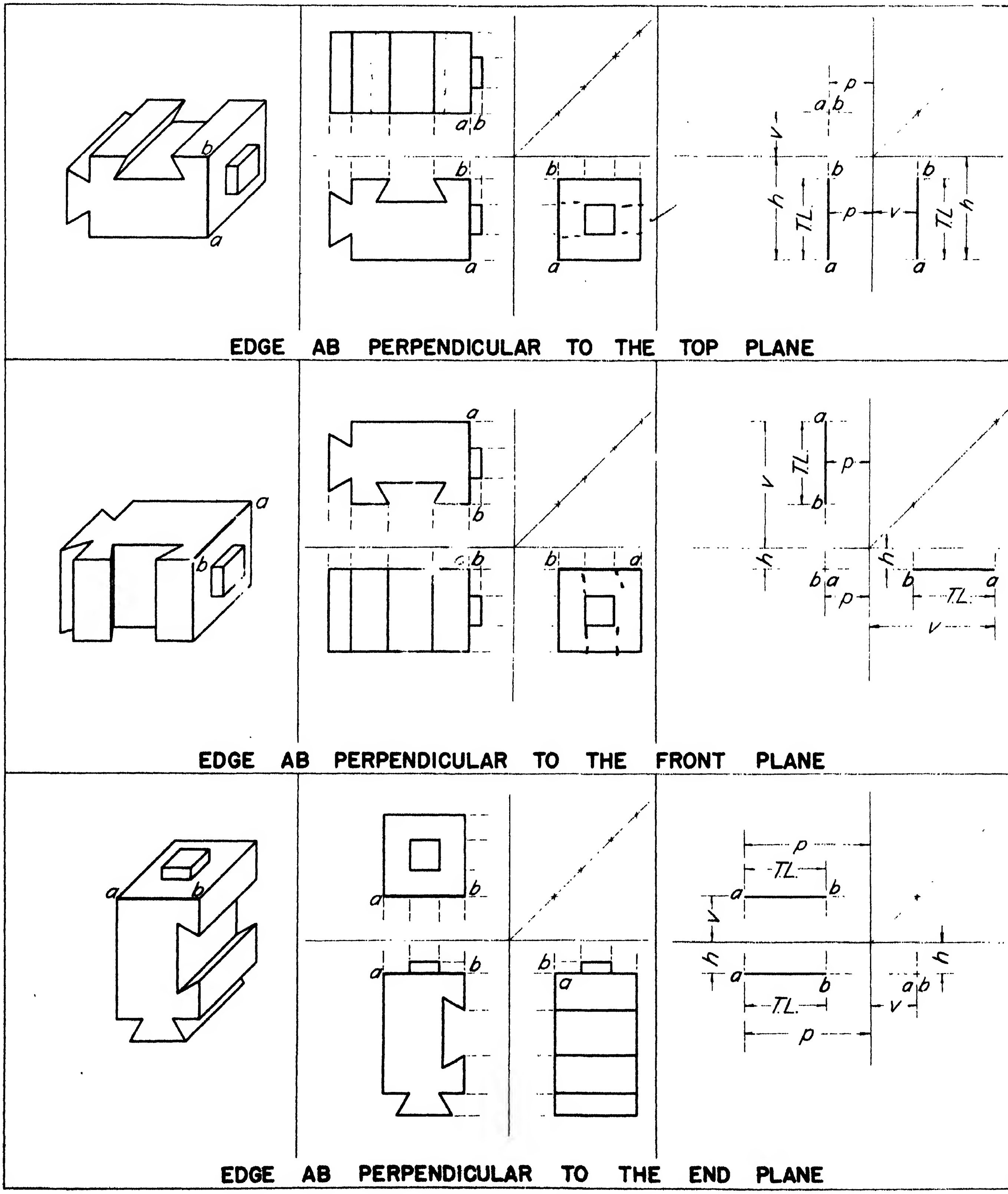


FIGURE 16a

Three possible line positions.

Distances from the three principal planes may vary, but these views represent the relationship of the lines to the reference planes in the positions described.

24. **Possible Line Positions** (*cont.*). The block in Fig. 16*b* has been placed so that its edge *AB* successively becomes *parallel* to the front, the top, and the end plane of projection. In all these graphic descriptions, it is to be observed that the view of edge *AB* on the plane of projection to which it is parallel is the *only view* which is equal in length to *AB*, and that all other views are *shorter*. Therefore, when *any view* of a line is equal in length to the *line itself*, the *line* must be *parallel* to that *reference plane* upon which *this view* of the line is *projected*.

In the drawings of Fig. 16*b*, only one slope of the line, of the many slopes possible, is shown. Other slopes would vary in the angle of inclination but would not affect the principles of representation illustrated.

Top Row. The edge *AB* inclines at an angle to the top plane but is parallel to the front plane. In the three-view drawing of this block *AB* appears, therefore, as a sloping line in the front view equal in length to *AB*, while both the top and right end view are shorter than the true length. In the drawing at the right, edge *AB* is shown by three views, each dimensioned to show the position and relationship of line *AB* to the principal planes of projection. It is to be noted also that the front view of the line shows the true shape and size of the angle of inclination to *H* since this view is a fundamental view.

Middle Row. The block has been turned so that edge *AB* is now parallel to the top plane and oblique to *V* and *P*. The three-view drawing shows, therefore, that only the top view indicates the true length of *AB* while the front and end views are foreshortened. In the drawing of edge *AB* as a line alone, the dimensions show the position and the relationship of *A* and *B* to the projection planes. The top view, it is to be noted, shows not only a fundamental view of *AB*, and hence its true length, but also the shape and size of the angle *AB* makes with the front, or *V*, plane.

Bottom Row. The block has been located to make its edge *AB* parallel to the right end plane and oblique to *H* and *V*. The three-view drawing of the block, therefore, shows *AB* and the whole face of the block of which *AB* is one edge in its true shape, size, and location below *H* and behind *V*. The drawing at the right, showing three views of *AB* alone, is dimensioned to illustrate how such a drawing locates the line with reference to the planes of projection, and shows true values. The end view shows not only the true length of *AB* but also the true values of the angles of inclination of *AB* to both *H* and *V*.

Suggestion: Check the projection by means of triangles, and note carefully the construction method for drawing by projection.

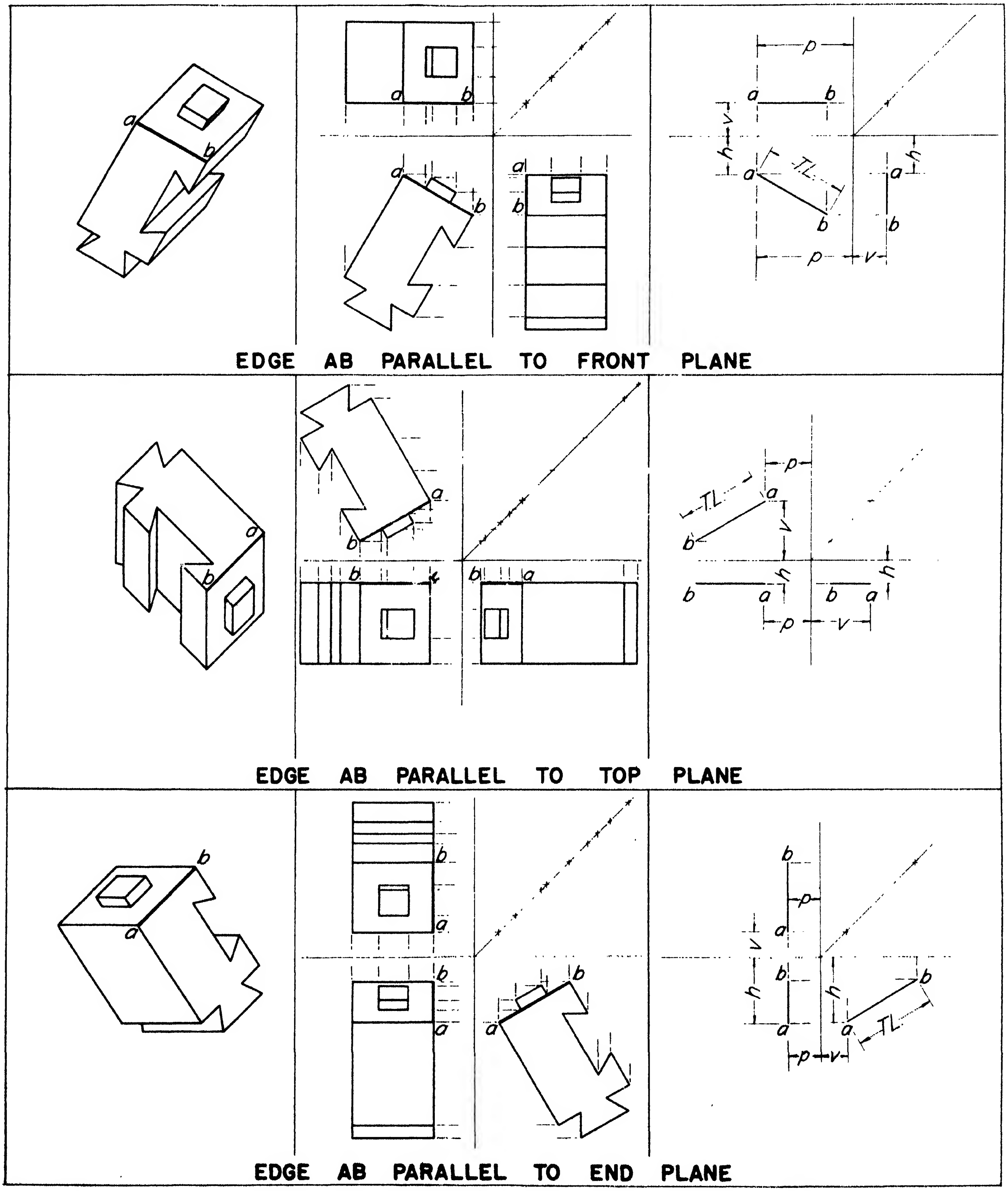


FIGURE 16b

Three possible line positions.

Distances from principal planes, and the degree and direction of slope, may vary, but these views represent line relationship to the principal planes when lines are in the typical positions described.

24. **Possible Line Positions** (*cont.*). In the six positions of the block illustrated and described in Figs. 16*a* and 16*b*, edge *AB* was parallel to some one of the principal planes. This line was also in the three cases described in Fig. 16*a* perpendicular to one plane as well as being parallel to two other principal planes. In all these six positions some *one view* of the line on a principal plane was a *fundamental view* from which all the essential facts about the line could be determined.

In Fig. 16*c*, however, the seventh position of a line is shown. The block has been so rotated that its edge *AB* is neither parallel nor perpendicular to any of the principal planes.

Top Row. The three-view drawing of the block shows clearly that no line, no face of the block is parallel or perpendicular to any of the principal planes. The three-view drawing of the edge *AB* alone shows this line, therefore, oblique to all three planes. Although no true information about length or slope is disclosed by this drawing, the position and relation of points *A* and *B* to these reference planes are shown. From this graphic information, a draftsman by using new views of the auxiliary type may derive information about true length, slope, and direction by utilizing the facts available on the drawing. The method for doing this will be explained in Article 26, Fig. 17.

Middle Row. The two three-view drawings of line *AB* represent a line sloping at a different angle in each case and in a different direction and illustrate how slope affects the position and appearance of the principal views. Although true length and the true angles of inclination to *H*, *V*, and *P* are not shown in measurable position in either of these drawings, enough graphic data are available on these drawings to enable a draftsman to find this information. While obviously an infinite number of positions for oblique lines are possible, the degree of obliqueness in no way affects the principles of representation involved in describing position with reference to principal planes of projection.

Bottom Row. In this graphic description of a line *AB* a new slope and position are described by four principal views. Again it will be observed that none of these views is a fundamental view, and, therefore, no true facts about length of angle of inclination appears in measurable form. To find such information the draftsman will be obliged to use the given data as the basis for locating and projecting a new view on an auxiliary plane of reference.

The fact that this is always true in dealing with oblique lines and planes emphasizes the need for a thorough understanding of fundamental views.

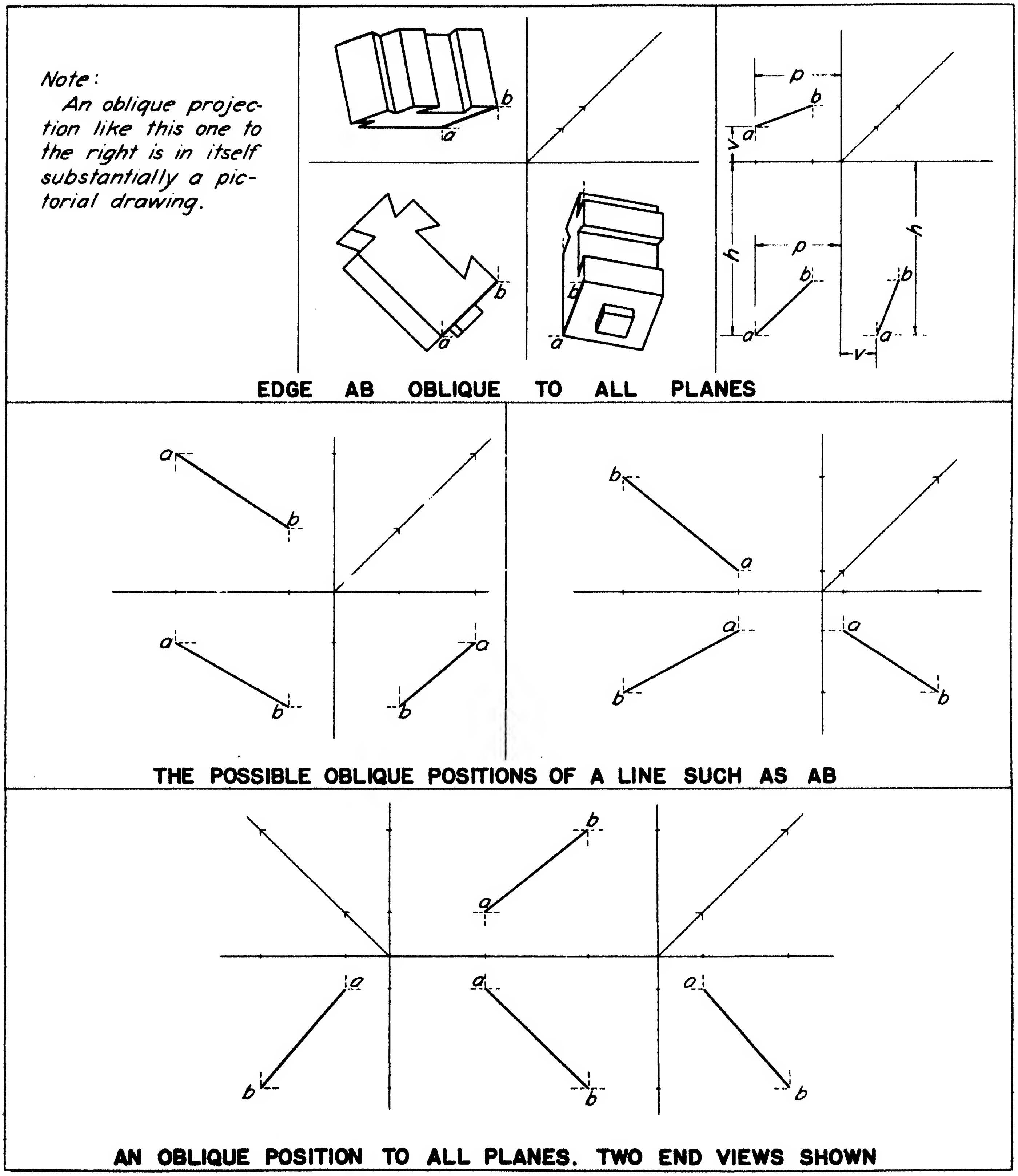


Figure 16c

The possible oblique positions of a line.
Distances from principal planes, the direction, and the degree of slope may vary, but these views show that oblique lines have views oblique to all principal planes of reference.

25. **Fundamental Views.** A fundamental view is that view which shows true relationship. A fundamental view, therefore, always is drawn on a plane, either a principal plane or an auxiliary plane, which is parallel to the relationship to be described. The fundamental view of a line shows its true length and slope; the fundamental view of a plane shows its true shape and size if the plane is bounded by lines, and if defined only by parallel or by intersecting lines its fundamental view will show the true relationship of these defining lines.

The three important points to know in the analysis of drawing-board geometry problems are: (1) what positions of lines, or planes, or combinations of points, lines, and planes, require auxiliary views for their complete description; (2) the position in which to place the auxiliary plane to secure the fundamental view which describes true relationships; (3) the projection methods used in securing fundamental views.

26. **To Find the Fundamental View of a Line.** In Fig. 17, the fundamental view of oblique line *AB* is found on two auxiliary planes: one plane (RL_h) is perpendicular to *H*, and the other (RL_v) is perpendicular to *V*, but both auxiliary planes are *parallel* to line *AB*.

In the top figure is given the construction for finding the auxiliary view of *AB* on these auxiliary planes by projection; in the bottom figure the auxiliary views are found by measurement and projection. The construction in the bottom figure is more commonly used.

Since line *AB* is oblique to all principal planes, an auxiliary view will be required to describe its length and slope. Since this auxiliary view must be a fundamental view the auxiliary plane must be located *parallel* to *AB*, and must be either perpendicular to *H*, the top plane, or to *V*, the front plane. (In the figure, both possible auxiliary planes are located.)

The geometric projection method and the combined measurement and projection method of locating the fundamental view of *AB* on these auxiliary planes are shown in detail on the drawing and may be comprehended by reading the drawing.

27. **Problem Models.** Figure 17*a* represents the identical line *AB* which was used to illustrate the derivation of the fundamental view of a line in Fig. 17. This page may be removed, cut, and folded as directed on the drawing, to make a model of the problem showing the actual relationship described by the drawing. A study of this model when folded and a comparison of the model with Fig. 17 will clarify and establish the relationship described by a drawing with the same relationships as shown in space on the model.

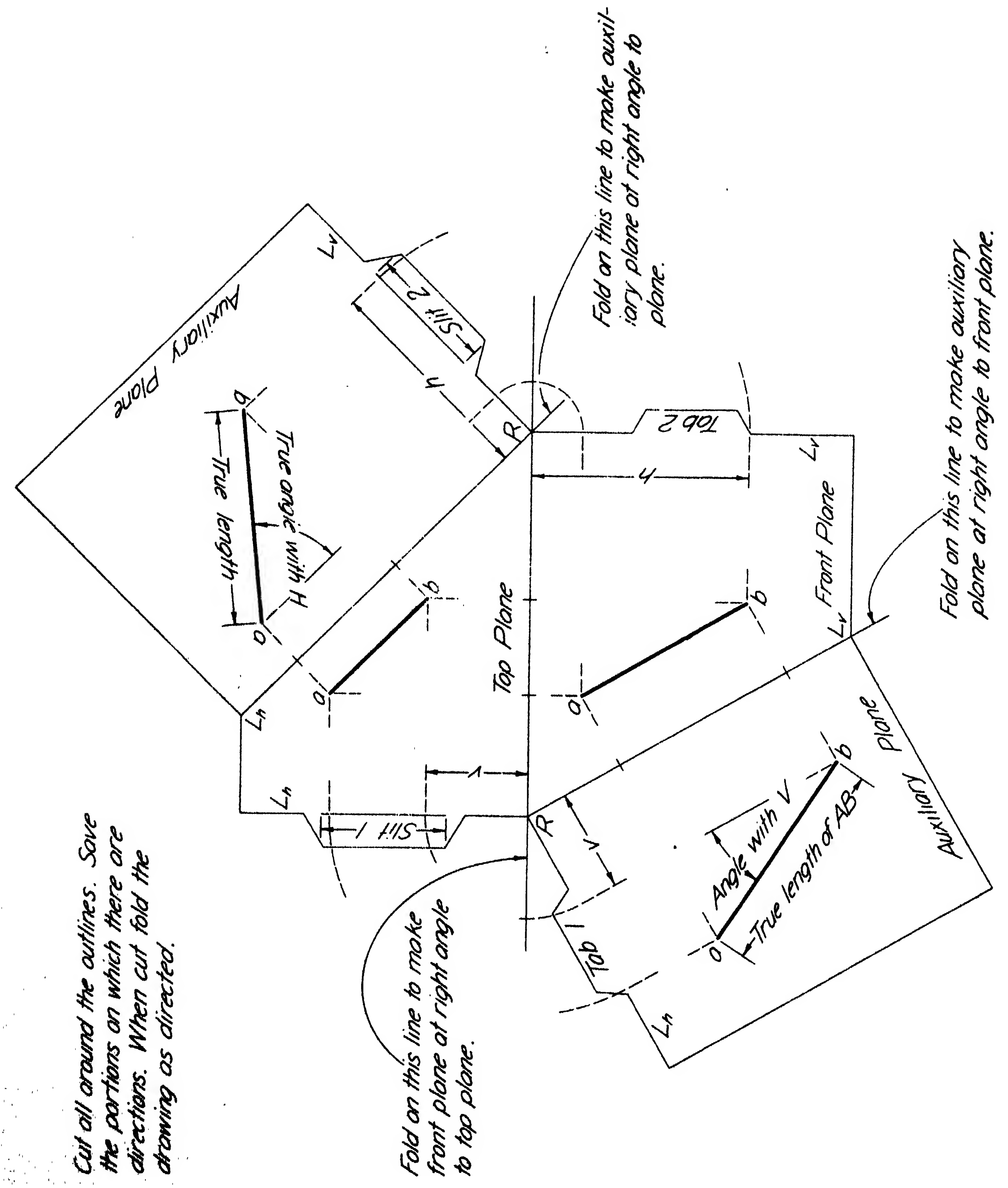


Figure 17a

Remove this page from the book. Cut and fold as directed and a model of Fig. 17 will be formed.

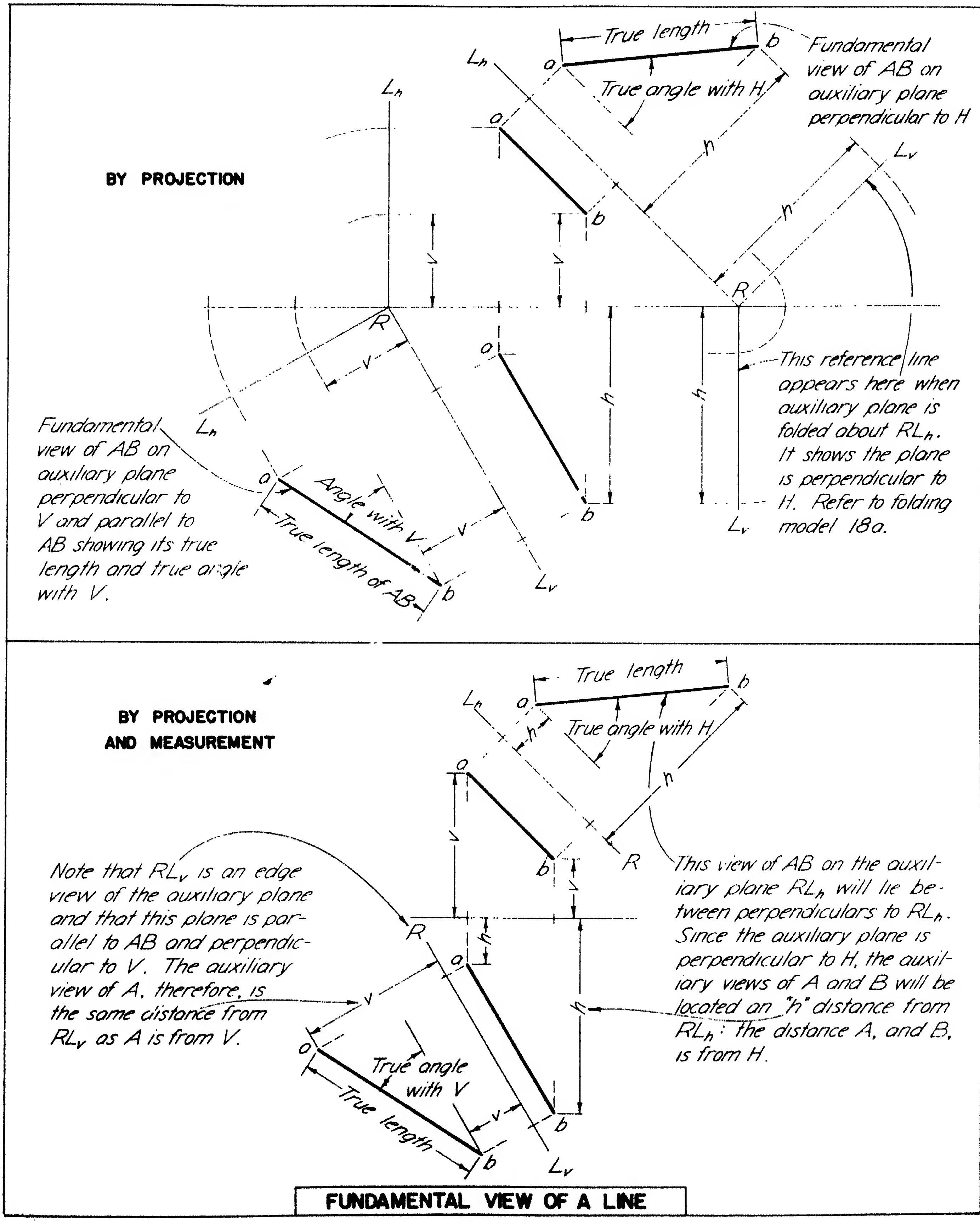


Figure 17
The derivation of a fundamental view of a line.

THE SCIENTIFIC METHOD

Problems involving relationships between points, lines, and planes, or involving relationships of these elements with surfaces, may be solved by "the scientific method" Refer to Articles 15 and 16.

SURVEY

Factual data about the problem is either collected, or is given in written form. These facts must be thoroughly understood. Refer to Article 15A.

DRAWING

The facts relating to the problem must be recorded in graphical form. By means of such graphical data the relationships are visualized. Refer to Article 15B.

ANALYSIS

By means of analysis, (a) the requirements of the problem and (b) how to represent and measure these data are discovered. Refer to Article 15C.

SOLUTION

Several methods of solution may be indicated by the analysis. The most direct and accurate method is, of course, the best. Such a method will be found in one or more of the Essential Principles. Refer to Article 15D.

THE ESSENTIAL PRINCIPLES

EP1 To measure the length, the direction, and the slope of a line. Article 30.
EP2 To locate a line of given length, slope and direction. Article 31.
EP3 To locate and measure the shortest distance from a point to a line. Article 33.
EP4 To measure the angle between two lines. Article 37.
EP5 To construct a plane figure of given shape, size and location. Article 39.
EP6 To find where a line pierces a plane. Article 40.
EP7 The principle of revolving. Article 41.
EP8 To locate and measure a perpendicular from a point to a plane. Article 43.
EP9 To locate and measure the shortest line connecting two lines which are neither parallel nor intersecting. Article 44.
EP10 To measure the angle a line makes with a plane. Article 45.
EP11 To locate the line of intersection between two planes. Article 46.
EP12 To measure the angle between two planes. Article 47.
EP13-19 Problems on surfaces. Articles 53-83 inclusive.

Essential Principles are based on the application of foundation knowledge. Without this basic knowledge neither will the principles be understood nor will it be possible to solve problems in Drawing Board Geometry. Refer to Chapter IV.

BASIC KNOWLEDGE

1. Orthographic view arrangement. Refer to Articles 6-12.
2 View derivation by orthographic projection. Refer to Article 23.
3 The purpose and derivation of auxiliary views. Refer to Articles 12-14, and 22.
4 The fundamental view of a line. Refer to Articles 25 and 26.
5 The fundamental view of a plane. Refer to Articles 35 and 36.
6 Perpendicular relationships. Refer to Articles 32 and 42.

INDEX OF PRINCIPLES

Index of Principles.

CHAPTER IV

ESSENTIAL PRINCIPLES

28. **Essential Principles.** The basic drawing-board operation required in solving problems of three-dimensional nature consists in finding a fundamental view. The method of finding the fundamental view of a line is described in Article 26 and Fig. 17; the method of finding the fundamental view of a plane will follow in Articles 35 and 36 and Figs. 24 and 25.

With a working knowledge of these fundamental operations, it becomes possible to solve the problems involving points, lines, planes, and surfaces directly on the drawing board by graphical methods. These problems are catalogued as essential principles and for convenience and brevity are listed as EP1, EP2, etc. They are arranged in such sequence that each depends to some extent on the one which preceded. For example; unless the derivation of the fundamental view is understood, and the drawing-board operations required for its graphical representation mastered, the first essential principle cannot be comprehended. In like fashion, Essential Principle 2 depends upon an understanding of EP1. Therefore, thoroughness in study and in understanding as the subject progresses is imperative.

Engineering students should constantly keep in mind, therefore, the fact that the study of drawing-board geometry is in reality itself an engineering undertaking demanding sound and complete foundation work before a superstructure is begun.

29. **To Measure Direction and Slope of Lines.** The direction of a line in drawing-board work is usually expressed by its bearing, which is its angular deviation from true north or south (as, for example: N30°–15′W, or S16°–17′E), or by its angular inclination to the vertical plane of reference (as, for example, 60° with V). The slope of a line is expressed as its angular inclination to the horizontal plane of reference (as 30° with H), or by its grade, which is the number of feet of rise or fall for each 100 feet of horizontal run (as, for example: 15 per cent rising grade, meaning that for each 100 feet of horizontal run the line rises 15 feet above the level of the starting point).

Although this book does not include all the problems of value and practical application, enough of the *essential principles* have been explained and demonstrated to equip the student to solve the usual problem situations which he will meet in the drafting room. With a knowledge of these principles, new and even unusual problems may be reduced by analysis to these fundamentals and successfully solved.

To find the direction of a given line: Direction is measured on the plan view. True north is always the top of a map (or it is indicated by a drawn north point). The direction of a line is stated by its bearing in degrees, minutes, and seconds east or west of true north or south. In Fig. 18*a*, *ab* is a given plan view, and on the drawing is indicated the method of finding its bearing of N36°–19′–00″E, the starting point being *a*. If *b* were taken as the starting point the bearing would be S36°–19′–00″W.

To lay off a line in a stated direction: A starting point must be given or assumed. In Fig. 18*b*, *a* is taken as the starting point of a line of indefinite length which is to bear N38°–40′–00″E. If the length of this line were given, this distance would be measured on line *ab* if it were the horizontal distance; if the length were the actual true distance a fundamental view would be required as explained later in Fig. 20.

To find the slope of a line: Slope is expressed either as the angle a line makes with the horizontal, or in grade. Grade, it should be observed, is in reality the natural tangent of an angle when its adjacent side is 100. To measure slope, a fundamental view of the line is required; such a view is given as *mn* in Fig. 18*c*. This figure shows not only the method of measuring the slope, but also the position of the angle with respect to the measuring lines.

To lay off a line of given slope: When slope is expressed by the grade of a line, a rising slope (or plus) indicates that the far end of the line is higher than the starting point; when a falling slope is indicated (or minus) the far end is lower than the starting point. In Fig. 18*d*, *m* is the starting point of a line which slopes on a —12 per cent (or falling) grade. This means, of course, that any point *n* taken 500 feet from *m*, measured on the horizontal, in the given direction will be 60 feet lower than *m*. It is obvious from the drawing that the actual connecting distance between *m* and *n* is more than 500 feet.

Draftsmen must keep in mind that in map work distances are measured and laid off on the horizontal. In other words, the actual length of a tunnel between two points at different elevations would be longer than the "map" or plan view. The plan view, therefore, shows the measured but not the ***true length*** of such a line connecting two points.

The most satisfactory method for measuring and laying off angles on the drawing board, both accuracy and tools being considered, is the method of natural tangents. The natural tangent of an angle is the value obtained by dividing the side opposite the angle to be measured by the side adjacent to the angle when these two sides form 90°. The values of natural tangents are available in handbooks and other convenient sources and for that reason are not included in this book.

In structural drawings, and drawings of like nature, slope is shown by a slope diagram which is based on the steel square. A right triangle is drawn with the hypotenuse parallel to the slope, the adjacent side 12 inches (to scale, of course), and the opposite side indicating the natural tangent on a base of 12.

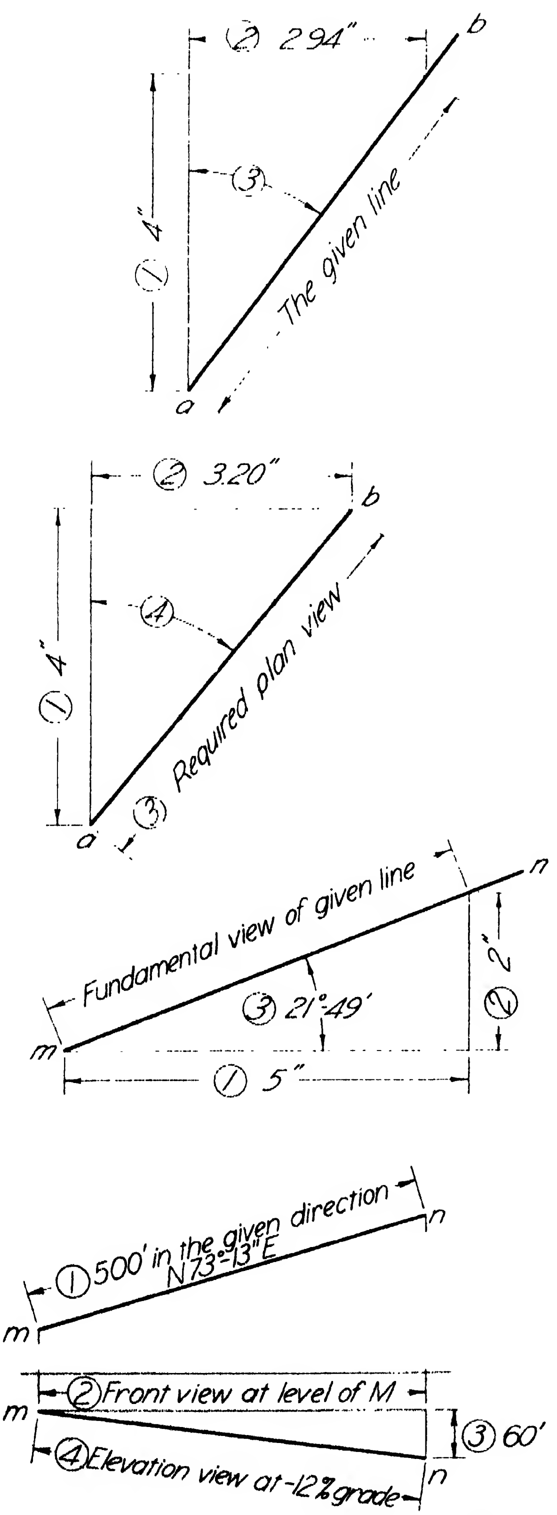


Figure 18a:- To find the direction of a given line.
(1) Through the starting point a draw a true north line. Make this any convenient length, say 4".
(2) Through this 4" point draw a second line at right angles to the north line. Measure the length of this line between the given line ab and the north line. This distance is 2.94". Divide this distance by 4 and locate the obtained value (0.735) in a table of natural tangents.
(3) The angular value (36°-19') will be the angular deviation of line ab from north, thus giving a bearing of N36°-19'E.

Figure 18b:- To lay off a line in a stated direction.
(1) Draw a true north and south line through the starting point a. Make this line any convenient length, say 4".
(2) Through this 4" point draw a second line at right angles to the first. Make this line 3.20" long. This value is obtained by finding in a table of natural tangents the natural tangent of the given angle 38°-40' and multiplying it by 4" the length of the adjacent side.
(3) Connect the point b (found as indicated in 2) with point a. This line ab inclines 38°-40' to north and bears N38°-40'E.
(4) The size and position of stated angle of direction.

Figure 18c:- To find the slope of a given line.
(1) Draw a horizontal line through the starting point m. Make this line any convenient length, say 5".
(2) Erect a perpendicular to the horizontal line at the 5" point. Measure the length of this perpendicular between line MN and the horizontal line. This line measures 2.0". Divide 2" by 5" and the quotient is the natural tangent (0.400) of the slope angle.
(3) Find the angular value in a table of natural tangents for the tangent 0.400. This value (21°-49') is the slope of line MN.

Figure 18d:- To lay off a line of given slope.
(1) Measure 500' horizontally in the given direction.
(2) Locate the end of 500' distance in the elevation view.
(3) At this point erect a perpendicular 60' long (note that 60' is 12% of 500') and locate n lower than m.
(4) Connect m and n in the elevation view thus obtaining the required view of line MN sloping downward at 12% grade.

TO MEASURE DIRECTION AND SLOPE OF LINES

Figure 18

The measurement of direction and slope of lines.

30. **EP1. To measure the length, the direction, and the slope of a line.**

The length of a line is obtained by finding any one of its several fundamental views (Article 26, Fig. 17) and measuring this view with the scale.

The direction (or bearing) of a line is obtained by finding the angle its top view deviates from true north or south and the direction of deviation (as east or west). (See Article 29, Fig. 18*a*.)

The slope of a line is measured by finding the angle the line makes with the top plane (Article 29, Fig. 18*c*) or by finding the grade of the line. (Article 29, Fig. 18*d*.)

The angle the line makes with V, or the front vertical plane, may be obtained by finding the appropriate auxiliary view of the line on an auxiliary plane perpendicular to V and measuring the fundamental view of the angle thus shown. (Fig. 19.)

1. **To find the direction of line *MN*.**

In Fig. 19, MN is a line shown by a top view and front elevation. The problem is to find its length; its slope or the angle it makes with H, the top plane; and the angle it makes with V, the front plane.

By drawing a north line through m in the top view the position and size and direction of the bearing may be observed at once. By obtaining the value of the bearing angle (Article 29, Fig. 18*a*) the direction of the line MN may be given as N35°–5′E. (See Fig. 19, note 1.)

2. **To find the angle *MN* makes with *H*.**

By locating an auxiliary plane parallel to MN and perpendicular to the top plane (see Fig. 19, note 2), the angle the line MN makes with the top plane, or the angle of slope, may be viewed in a normal direction, and when projected upon the auxiliary plane indicated by RL_h its fundamental view is obtained and may be measured for its value by the method illustrated in Article 29, Fig. 18*c*. (See also Article 26, Fig. 17.) The angle in this fundamental view having been found and measured, the slope of the line may be given as 44°–18′ with the top plane.

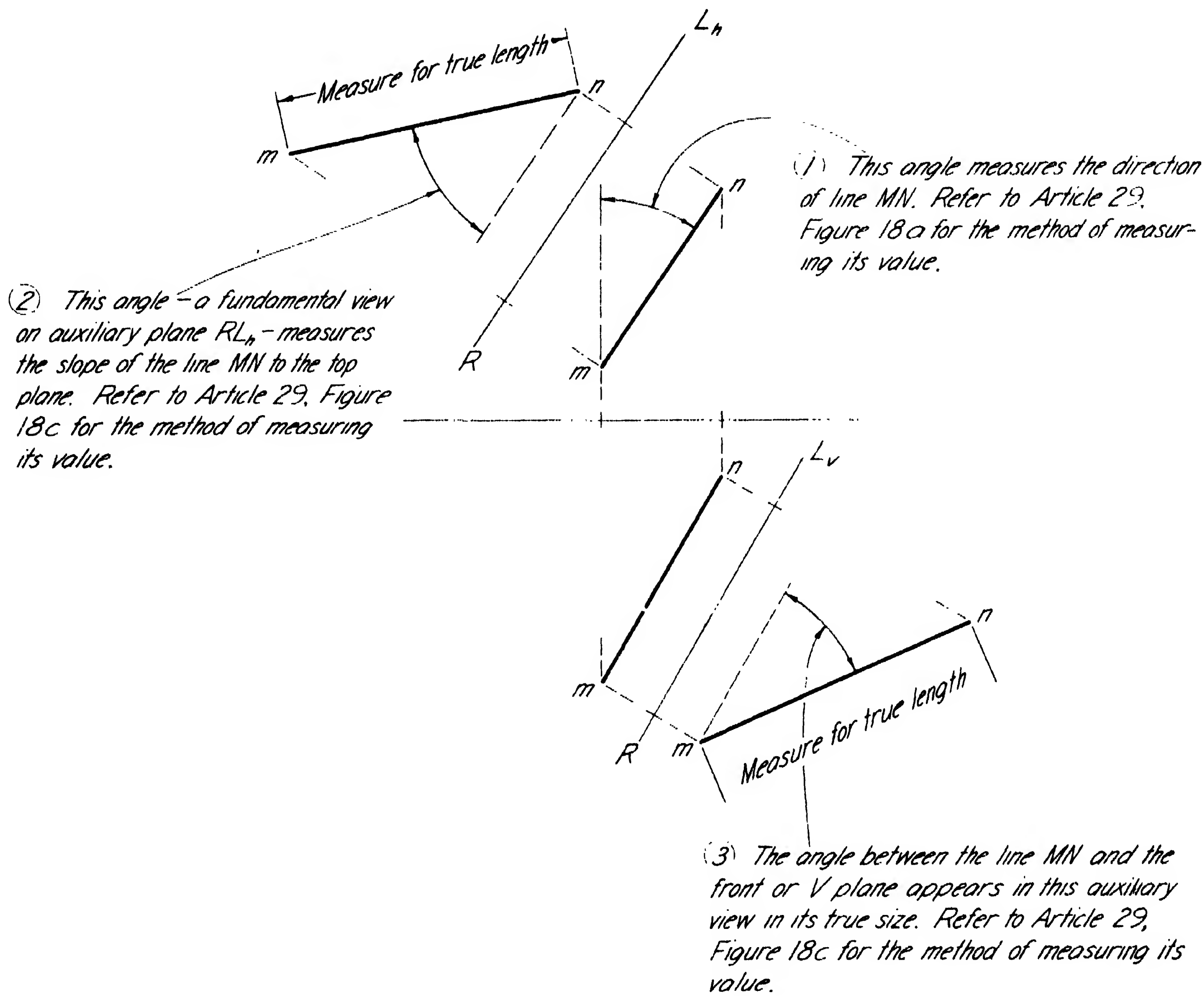
3. **To find the angle *MN* makes with *V*.**

By locating an auxiliary plane (RL_v) parallel to line MN and perpendicular to the front plane V (see Fig. 19, note 3) a fundamental view may be obtained (Article 26, Fig. 17) which will show in true size and relation to V the line MN. By measuring the value of this angular relationship, the value of the angle MN makes with V may be stated as 36°–26′.

To measure the length of *MN*.

Since both the view of MN on RL_h and on RL_v are fundamental views, these two auxiliary views of MN may be scaled and will, of course, be found equal, and will be equal to the true distance from M to N.

The value of the angle a line makes with *H* should be measured on the auxiliary projection of the angle on the auxiliary plane parallel to the line and perpendicular to *H*, similarly the angle a line makes with *V* should be measured on the auxiliary projection of the angle on the auxiliary plane parallel to the line and perpendicular to *V*. It is to be noted that these angles are *not* complementary.



Special Note:

Observe that MN is oblique to all principal planes and that therefore, neither the top nor the front view shows its true length.

Observe that since MN is oblique to H and to V, that neither the angle MN makes with H or V appears in the given views in its true slope or size.

Observe that the fundamental view on RL_h shows both the true length of MN and the true size of the angle MN makes with H; and that the fundamental view on RL_v shows both the true length of MN and the true size of the angle MN makes with V.

TO MEASURE LENGTH, SLOPE, AND DIRECTION OF A LINE

Figure 19

Essential Principle 1.

31. EP2. To locate a line of given length, slope, and direction.

Let the problem be stated as follows: From the bottom of a given shaft, A, locate the center line of a tunnel which runs north 32°–16′ west from A on a rising 12 per cent grade for 367 feet 4 inches measured along the tunnel.

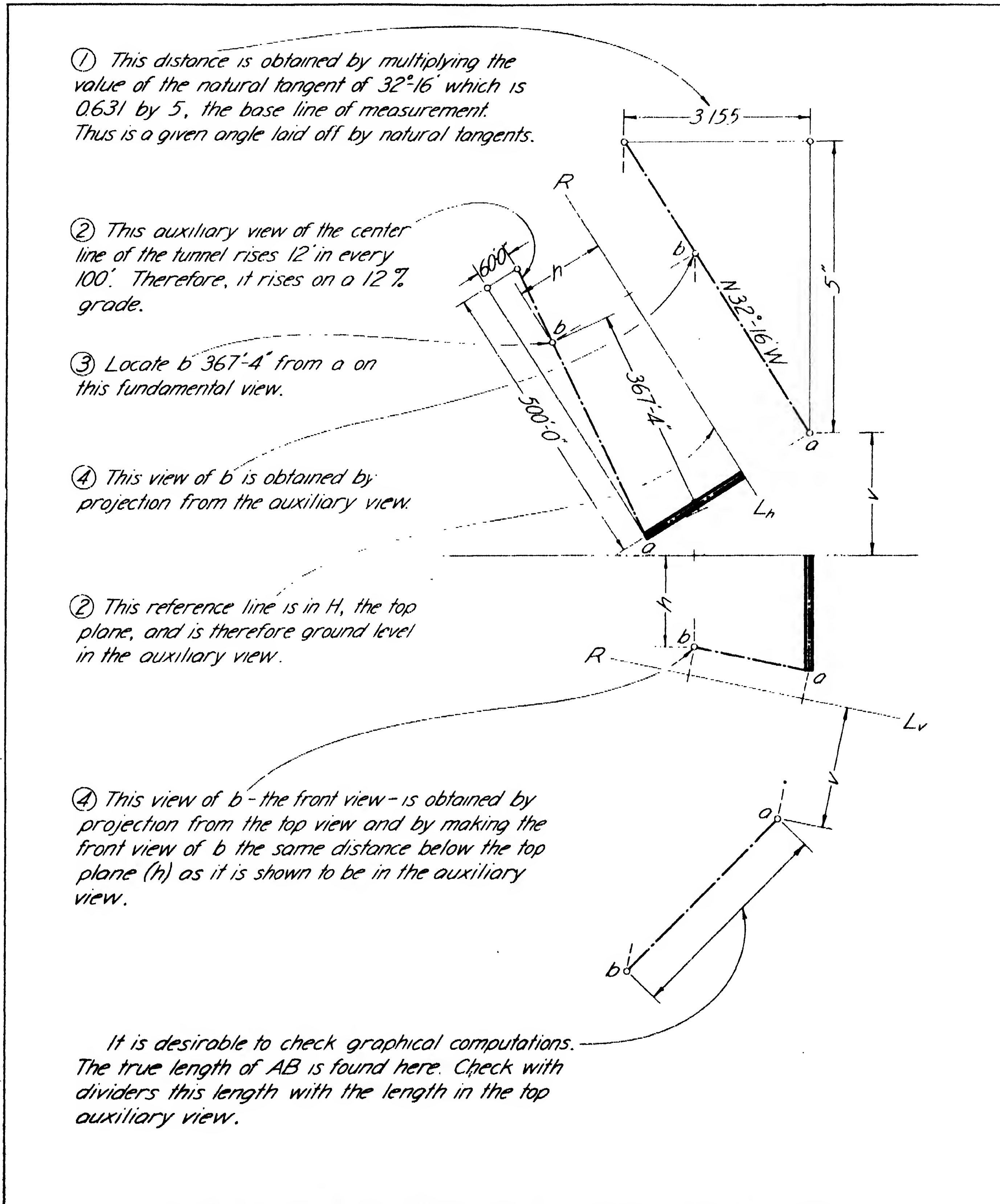
1. **Lay off the direction on the top view.** This may be done by laying off say 5 inches on a line true north from top view of A. From a table of natural tangents find the value of the tangent of 32°–16′. Multiply this value by 5 (since 5 is used as a base in place of the 1 value in the tables), and lay this value off by scale on a line perpendicular to north in the west direction. Through the west end of this line draw from a in the top view the center line of the tunnel, and let it be indefinitely long.

2. **Lay off the grade on an auxiliary plane perpendicular to the top plane and parallel to the center line.** Lay off 500 feet on a horizontal line through the auxiliary view of A—such a line will be parallel to RL_h—and erect a perpendicular 60 feet long (60 feet is 12 per cent of 500 feet). Through the 60 foot point and the auxiliary view of a draw the auxiliary view of the center line. This center line will be indefinitely long on a rising 12 per cent grade, since the line rises 12 feet in every 100 feet of horizontal run.

3. **Lay off the length on the fundamental view.** The auxiliary view of the center line just located will be a fundamental view. On it lay off 367 feet 4 inches from a. Thus find the fundamental view of ab which is the given length of the center line.

4. The point b may now be located by projection in the top view, and in the front view. Thus is located line AB, the center line of given length, of given slope, and of given direction.

By the application of similar methods any line may be located, provided that length, slope, and direction are known. The principle involved is to find the fundamental position of the line from the data given.



TO LOCATE LINES OF GIVEN LENGTH, SLOPE, AND DIRECTION

FIGURE 20

Essential Principle 2.

32. **Perpendicular Relationships of Lines.** When two lines are at a right angle to each other, such a space relationship is readily understood. The draftsman's problem is one of correctly representing such a relationship by views on a drawing.

When a line is shown by a *fundamental view*, any line *perpendicular* to it will be projected on the reference plane of this *fundamental view* as a *view making 90°* with the *fundamental view*. This is an important projection principle which must be clearly comprehended. By its application, perpendiculars may be erected to a given line either from a point on the line, or from a point not on the line.

To erect a perpendicular to a given line at a point on the line. In Fig. 21*a*, a line *AB* is given by a top and front view. The line is parallel to *V*; therefore, the front view of *AB* is a fundamental view.

1. To erect a perpendicular to line *AB* at a given point *O* on the line, draw a line *O*1 making 90° with the fundamental view. Observe that *O*1 may be in *any position* or of *any length* so long as its front view inclines 90° to the front view *ab*.

2. *O*2, for example, is a second possible position.

3. To make this perpendicular any required length, or to locate it in any specified position, an auxiliary view on plane RL_v which is perpendicular to the line and to *V* is required.

To erect a perpendicular to a given line from a point not on the line. In Fig. 21*b*, *MN* is the given line to which a perpendicular is to be drawn from given point *O*.

1. Since neither the given top view nor front view of *mn* is a fundamental view, a fundamental view must be located on such an auxiliary plane as RL_v parallel to *MN* and perpendicular to *V*. (See Article 26, Fig. 17.) On the same auxiliary plane find the auxiliary view of the given point *O*.

2. On this fundamental view of line *MN*, erect a perpendicular to *mn* from *o* and thus locate *p*, the foot of the perpendicular.

3. Find by projection the location of *P* on the front view, and from this front view *p* the top view of *p* on the top view *mn*. Thus is located the top and front view of the required perpendicular from *O* to *MN*.

4. If the true length of *OP* is desired, locate an auxiliary view of *OP* on an auxiliary plane *RL* perpendicular to RL_v and parallel to *OP*. This fundamental view shows the length of *OP* in measurable form and also that *OP* is perpendicular to *MN*.

General perpendicular relationships as shown by a drawing. Figure 21*c* represents an oblique line *MN* bisected at *O* by a perpendicular *AB*.

1. It is especially to be noted that so long as the fundamental view *mn* makes 90° with the view *ab* on the same auxiliary plane, and the top and front views of *AB* are derived from this auxiliary view *ab*, *AB* and *MN* are mutually perpendicular.

2. *AB*, in other words, will be perpendicular to *MN* in any of the innumerable positions of the diameter of a circle made by rotating *AB* about line *MN* in a perpendicular plane of rotation.

3. When *AB* is parallel to V (a_2b_2) the front view of *AB* is a fundamental view and the perpendicular relationship shows in this front view in its true shape and size.

4. It also is to be noted that, when *AB* is parallel to *H* (a_1b_1), the top view is a fundamental view of *AB* and the top view of the right angle shows as 90°.

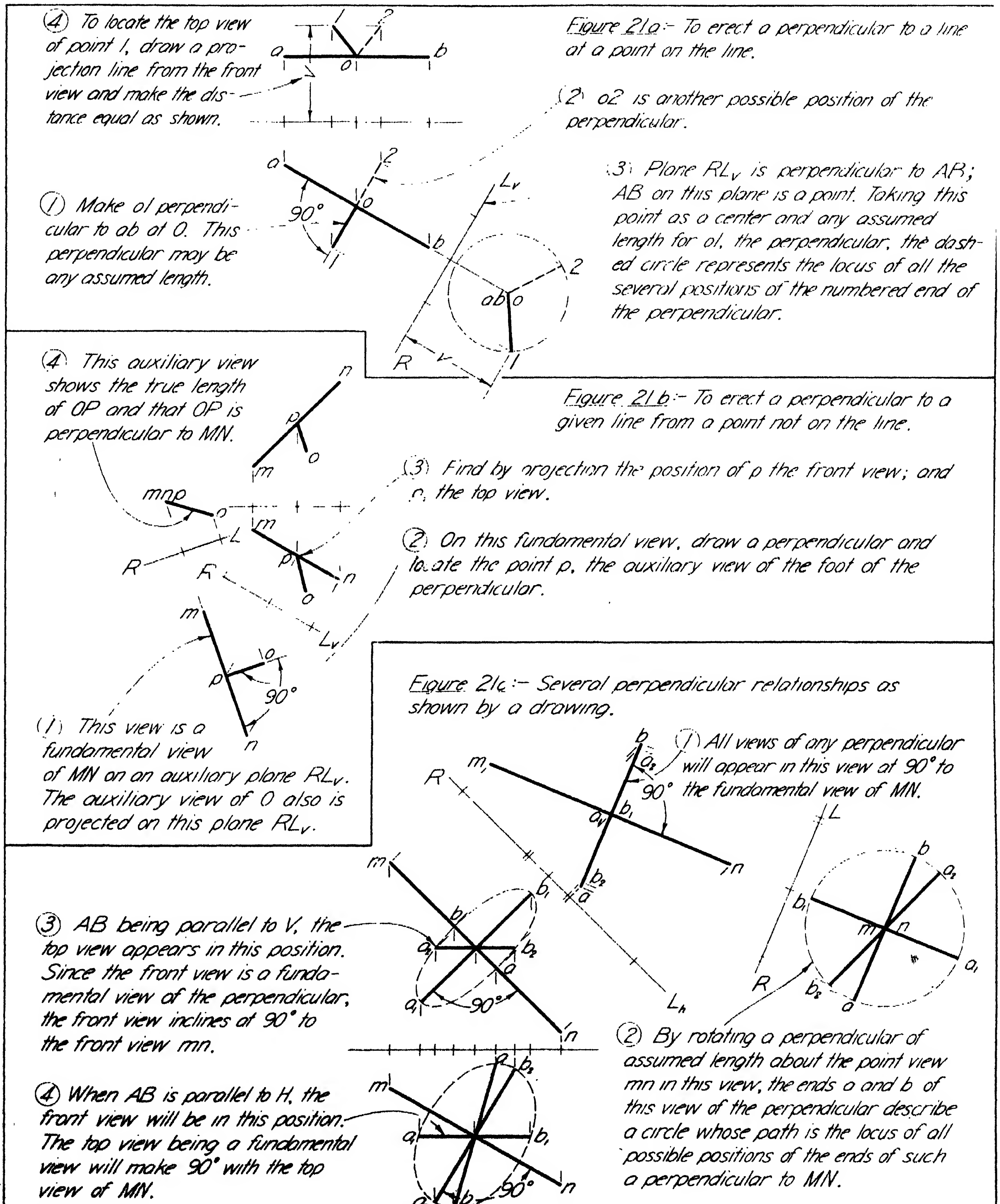


FIGURE 21
Perpendicular relationships of lines.

33. **EP3. To locate and measure the shortest distance from a point to a line.**

Obviously such a distance must be the perpendicular distance from the point to the line. The principle might also be stated: to locate and measure a perpendicular from a point to a line. The basic perpendicular relationships between lines are explained and described in Article 32, Fig. 21, to which reference should be made.

To locate and measure the shortest distance from a given point *O* to a given line *AB*. In Fig. 22 the given line *AB* is oblique to *H* and *V*, and is shown in relation to given point *O* by a plan and elevation view.

1. Locate an auxiliary plane RL_v parallel to the line *AB*, and find the fundamental view of *AB* on this plane. Also locate the projection of given point *O* in this auxiliary view.

2. Erect a perpendicular from this auxiliary view of *O* to the fundamental view of *AB*. Locate *P*, the foot of the perpendicular, on line *AB*.

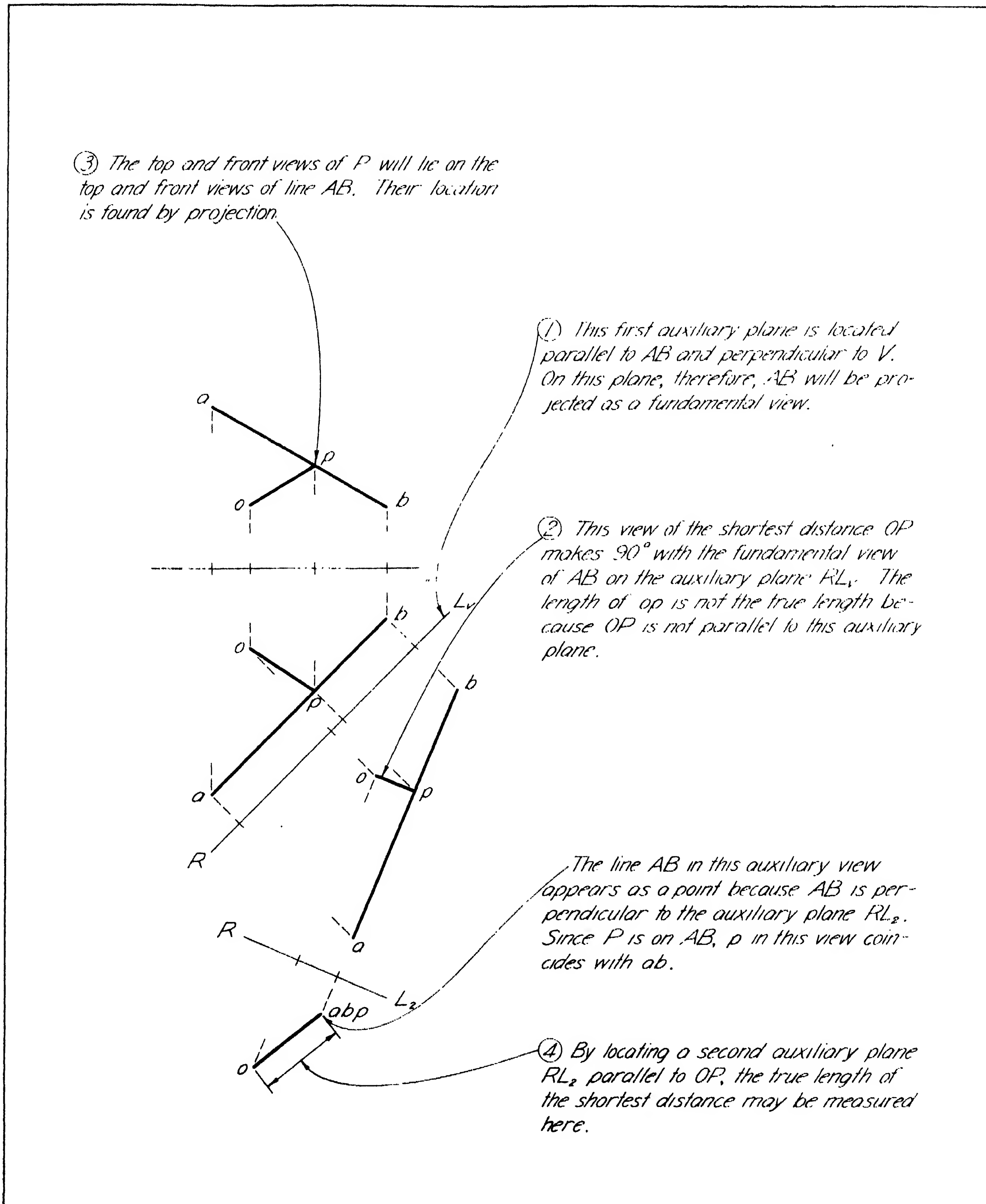
3. Since *P* is on line *AB*, the top and front views of *P* will lie on the corresponding views of *AB*. Therefore, locate the top and front views of *P* by projection and draw the top and front views of perpendicular *OP*.

4. To find the true length of *OP*, locate a second auxiliary plane RL_2 parallel to *OP* in its first auxiliary position. The view of *OP* on plane RL_2 will be a fundamental view and may be measured to find the true length of *OP*.

Note also that plane RL_2 is perpendicular to line *AB* in its first auxiliary position and that, therefore, the view of *AB* on plane RL_2 is a point (*abp*), and that the perpendicular relationship between *OP* and *AB* is proved.

Double auxiliary views. The use of a second auxiliary plane of reference is not uncommon and gives rise to the term "double auxiliary projection." The principle of auxiliary views is based on the theory that an object, or a space relationship, may be viewed from *any chosen position.* The *direction* of viewing is always *perpendicular* to the *edge view* of the *auxiliary plane*, and this edge view is shown by a reference line (*RL*). The *position* and *direction* of this auxiliary plane of reference are chosen by the draftsman to give a fundamental view of the relationship being studied, and such reference planes are always *perpendicular* to a *reference plane* already established. RL_2, for example, is perpendicular to RL_v, and its position and direction are chosen to describe a fundamental view of *OP*, derived from the view of *OP* on RL_v.

This relationship may be comprehended easily if all the parts of Fig. 22 are masked out except the views on RL_v and RL_2. If RL_2 were now made a horizontal line, these two views could be considered a plan and elevation view of the problem.



TO MEASURE THE SHORTEST DISTANCE FROM A POINT TO A LINE

FIGURE 22

Essential Principle 3.

34. **The Representation of Planes.** Planes usually appear in drawing-board work as bounded by outlines. Therefore, when two or more views of these outlines, or edges, are drawn, the position and location of the plane are described. Planes are also fixed as to position and location (1) by two lines which intersect; (2) by two parallel lines; (3) by three points. To establish the position and location of a plane, then, requires the boundaries of the plane, two intersecting or two parallel lines of the plane, or three points of the plane to be located with respect to the principal planes and two or more views of these elements shown.

In Figs. 23, an object is represented which has one plane face in all the possible positions a plane may assume. This plane face is bounded by a circle, it has a square opening in it, and two intersecting center lines. Thus all the conditions fixing the location and description of a plane are illustrated graphically.

Top Row. Figure 23a. The circular plane face is *parallel* to the front plane. The front view, therefore, is a fundamental view and shows the true shape and size of this face. The top and right end views of this plane appear as edge views. In the right-hand drawing where only the plane is represented, the projection relations and the distances the plane is from the planes of reference can be observed.

Middle Row. Figure 23a. Since the plane of the circular face is parallel to the top plane, the top view therefore is a fundamental view showing its true shape and size. Since the plane of the circular face is *parallel* to the *top plane* of projection, this circular face will be perpendicular to both the front and end planes. Its views therefore on these planes will be edge views showing the face as a line equal in length to the diameter of the circular face.

Bottom Row. Figure 23a. The position of the object has been changed so that the circular face is now parallel to the right end, or profile plane. The end view now shows the true shape and size of the circular face since this is a fundamental view. The geometrical relation between the three principal planes makes it obvious that, since the circular face is *parallel* to the end plane, it will necessarily be perpendicular to the top and front planes. The top and front views of the circular face will, therefore, be edge views, and in these two views the circular face will be described by a line equal in length to the diameter of the circle. The projectional relations and distances from the reference planes may be observed in the right-hand drawing where the circular face alone is represented.

In studying the representation of planes and their relations to the planes of projection, as well as other principles set forth in this book, it is important to read the text matter and graphic presentation simultaneously. By such methods of study, and by referring back and forth from written text to the drawn description, an understanding will be gained of how *views* of an object describe the *shape* of the object, and how by means of such orthographic views complete information may be obtained when it is not available elsewhere.

Furthermore, the drawings are drawn to scale and are orthographic projections. The graphic statements, therefore, may be checked by use of dividers and by applying the triangles to the drawing. By such checking an understanding of the derivation of views by projection principles will be facilitated.

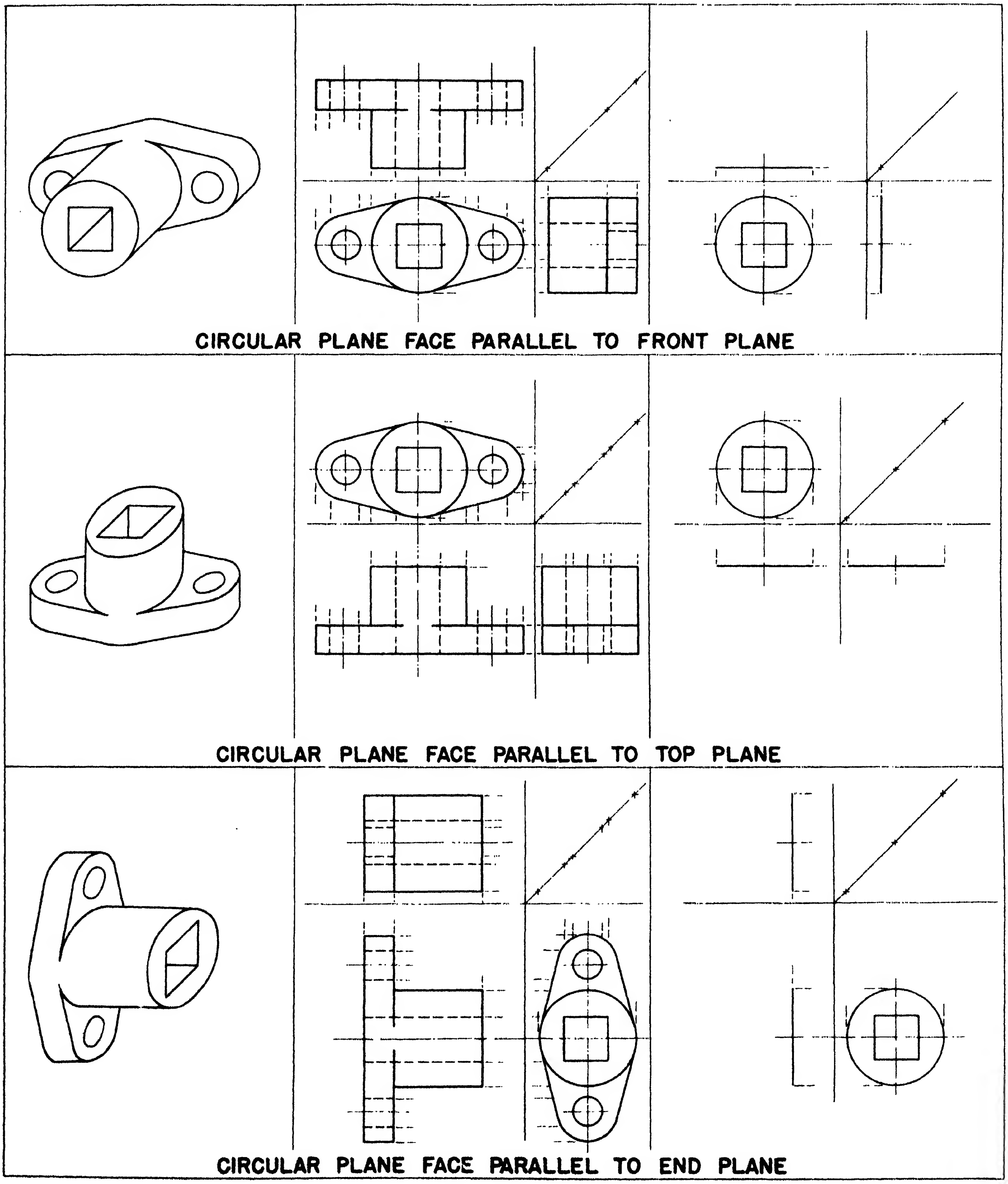


FIGURE 23a

Three possible positions of a plane.

The circular plane face may vary in distances from the principal planes, but the views will nevertheless have the same relations to the reference lines for each of the described positions.

34. **The Representation of Planes** (*cont.*). In Fig. 23*b*, the object has been turned so that the circular face is not parallel to any of the principal planes. This face is now placed so that it is perpendicular to each of the principal planes in turn, and at the same time oblique to the other two principal planes. It will be observed from the drawing, therefore, that none of the views in this entire series is a fundamental view because the circular plane face is parallel to none of the principal planes.

Top Row. Figure 23b. The object has been located so that the circular face is perpendicular to the front plane. The front view, therefore, is an edge view and is represented by a line equal in length to the diameter of the circular face. Note that this edge view shows the angle of inclination of the circular plane face both to H, the top plane, and to P, the end plane. Note also that since the circular plane face is not parallel to H, or to P, these two views must be derived by projection as shown. Since both these views are ellipses, the axes of these may be derived by projection and the ellipse constructed by the trammel method.

Middle Row. Figure 23b. The circular plane face in this figure has been placed perpendicular to H, or the top plane. The plan, or top, view therefore is an edge view and shows the angle at which the plane of the circular face inclines to V and to P. Owing to this inclination, both the front view and the right end view will be "foreshortened" and by projection will be found to be ellipses. The axes of these ellipses may be found and the view drawn by geometrical principles of ellipse construction.

Bottom Row. Figure 23b. The circular plane face in this illustration is perpendicular to the right end, or profile, plane, and shows, therefore, as an edge view whose inclination to the reference lines is the measure of the inclination of the plane face to H and V. Since the plane of the circular face inclines to H and V, its views on these planes will not be fundamental views but will be foreshortened according to the angle of inclination. Since this angle is 45° in this case both views will be alike in shape and size and must be located and drawn by projection.

Draftsmen must be alert to note and quick to take advantage of situations on the drawing board which enable them to arrive at a solution with a minimum of construction work. The projection of the top and front views in this last problem is a case in point. To construct these two ellipses from the readily drawn end view of the circular face would be a laborious process if projected point by point in the two views. By constructing these two ellipses by trammels, or by any adequate geometric method, from the easily derived axes involves much less construction and is likely to give a more accurate curve.

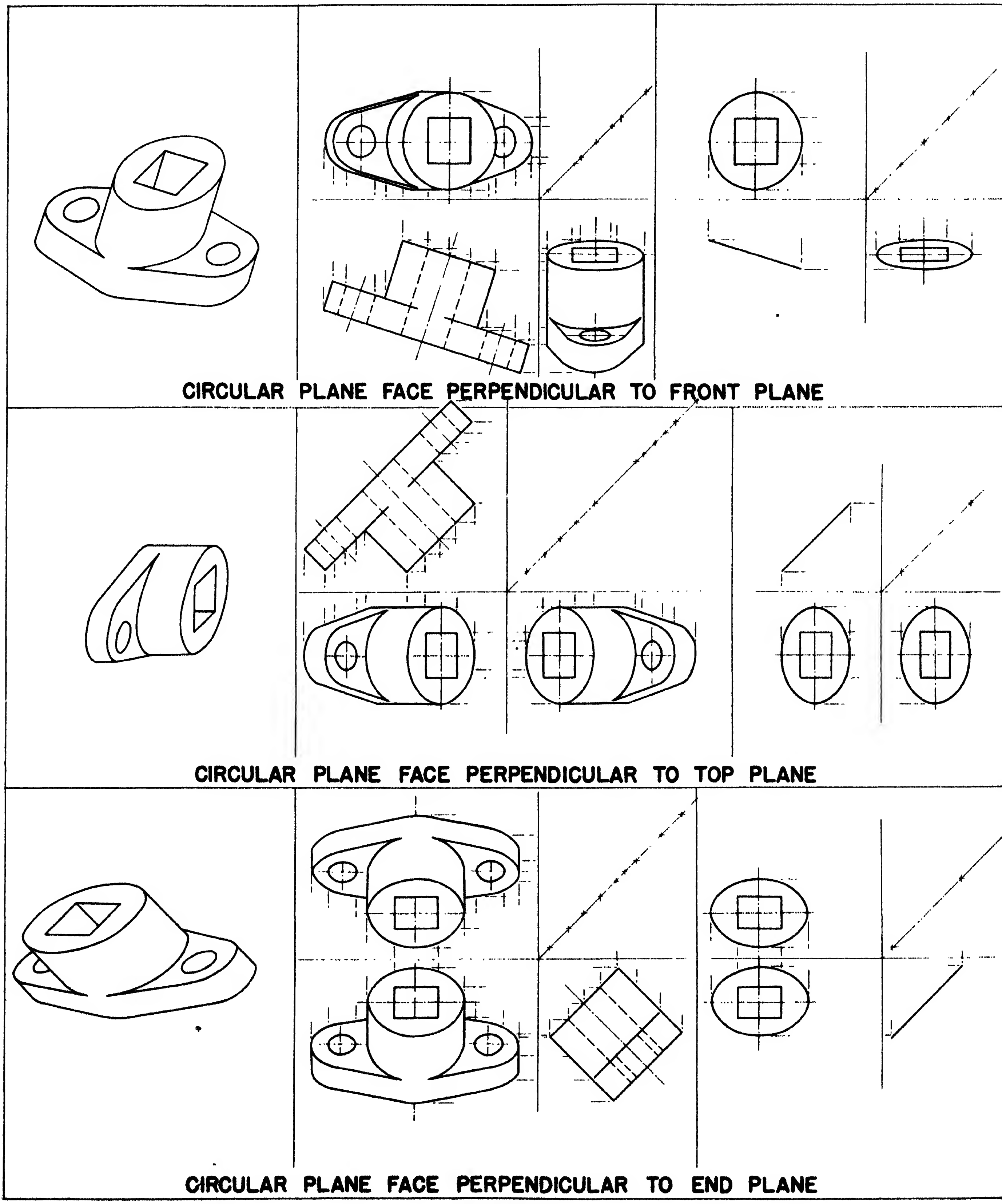


FIGURE 23b

Three possible positions of a plane.

The circular plane face may vary in distances from, and in degree of inclination to, the principal planes, but the views shown are typical for the positions described.

34. **The Representation of Planes** (*cont.*). In Fig. 23*c*, top drawing, the circular face of the object has been placed so that it is neither parallel nor perpendicular to any principal plane; therefore this face is oblique to all principal planes and appears on none of them as a fundamental view. In order to locate and draw a plane of this sort, either the location of the center lines, or of the lines of the square opening, or any three points of the plane must be given. Having such data, either in graphic form or by written description, a draftsman is able to locate by projection the three principal views as shown at the right. If, for example, one diameter is parallel to H, three views of this line may at once be located; since the top view of this diameter is a fundamental view, a top view of the diameter perpendicular to the first may be located and the front and end view of this line derived by projection provided that its slope to H is given. From these two perpendicular diameters as a base, the three views of the circular face may now be derived. Since it is known in advance that these three views are to be ellipses, by locating a fundamental view of one diameter in each view the axes of these ellipses may be found and the curves constructed by geometric methods.

The possible oblique positions of planes. The four figures in the lower drawing of Fig. 23*c* show all possible positions (except as to degree of slope) in which a plane may be when not parallel to one of the principal planes of projection.

Three of these positions show the circular plane as an edge view; and, therefore, the plane of the circular face will be perpendicular to the principal plane on which this edge view is drawn. In these three illustrations, it is to be noted that no view of the circular face is a fundamental view, but by locating a second reference plane parallel to the circular face a fundamental view may be derived. The *RL* for such a plane would be placed parallel to the edge view.

In the fourth drawing of this series, the circular plane face is oblique to all principal planes. To find the true shape and size of the circular face from a graphic description of this kind, it would first become necessary to find an edge view of the plane of the circular face and second from this edge view to derive a fundamental view.

The method of finding two such views is known as "double auxiliary projection" (see Article 33, Fig. 22) and is an important principle used in engineering drawing for the design and drawing of faces of an object when these are not shown in true shape and size.

The first and most important step in this method is to discover in which direction to view a plane face in order to project it on an auxiliary plane as an edge. This direction of viewing is indicated by the reference line (*RL*) of the auxiliary plane. In order to fix this direction so as to secure an edge view a principle called the "direction line method" is used. This principle and the method of applying it is explained in Article 36, Fig. 25.

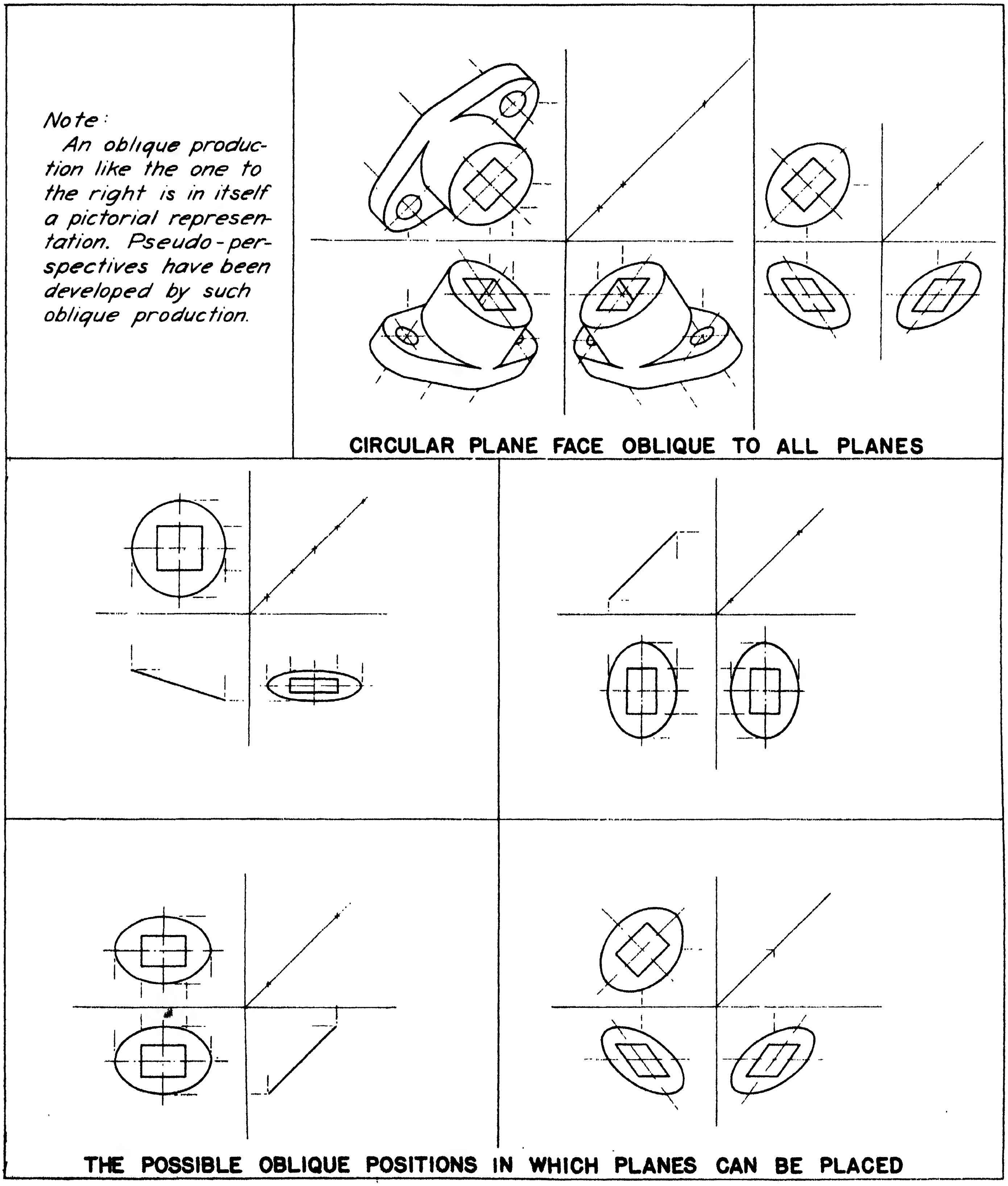


FIGURE 23c
The oblique position of a plane.

The distances from, and the degree of inclination to, the principal planes, may vary. This will affect the position, the shape, and the size of the views.

35. **The Fundamental View of a Plane.** In the upper drawing of Fig. 24, a cylinder, with a square hole through it, having a slanting face is shown by a top, a front, and a left end view. The slanting plane face of the cylinder appears in the top view as an ellipse, only partially visible, and in the front view as an edge. To show the true shape and size of this slanting face will require an auxiliary view.

To draw such a fundamental view, locate RL_v parallel to the edge view. This position of RL_v means that the auxiliary reference plane is parallel to the plane of the slanting face and also perpendicular to the reference plane on which this slanting face is drawn, and that the slanting face is to be viewed through this reference plane by an observer looking in a direction perpendicular to the auxiliary reference plane.

1. Locate on the auxiliary plane by projection from the front view and by measurement from the top view enough points to draw the auxiliary view of the cylinder including its slanting face. This view of the slanting face is a fundamental view and shows its true shape and size.

Note that, in drawing the ellipse in the auxiliary view, its axes were found by projection as indicated and the curve was constructed by the trammel method of drawing an ellipse.

In the lower drawing, Fig. 24, a rectangular truncated prism with a circular hole through it is described by a top, a front, and a right end view. The truncated face of the prism appears in the top view as an edge.

To draw a fundamental view of this slanting face, locate RL_h parallel to the edge view. In this case the auxiliary plane represented by its edge view RL_h is perpendicular to H, the top plane. The fundamental view showing the true shape and size of this slanting face will be seen by looking in a direction perpendicular to the auxiliary plane RL_h, and the fundamental view will be represented as a projection on this auxiliary plane. In the drawing, of course, the auxiliary plane has been rotated about RL_h, its edge, into coincidence with H so that the projection may be seen. The distances the several points of this rotated view are from RL_h are shown by the dimension h and distances parallel to h.

Partial Auxiliary Views. The auxiliary view in the lower drawing of Fig. 24 is known as a "partial auxiliary." Such a view is commonly used in engineering drawing in preference to a complete auxiliary as shown in the top drawing of Fig. 24. The purpose of auxiliary views in general is to add descriptive information to a drawing, and since only the fundamental portion of the auxiliary view in this drawing does that, the remainder of the auxiliary view is outlined in part only. The "break line"—the freehand irregular line—terminating this view is the conventional symbol which indicates a partial view.

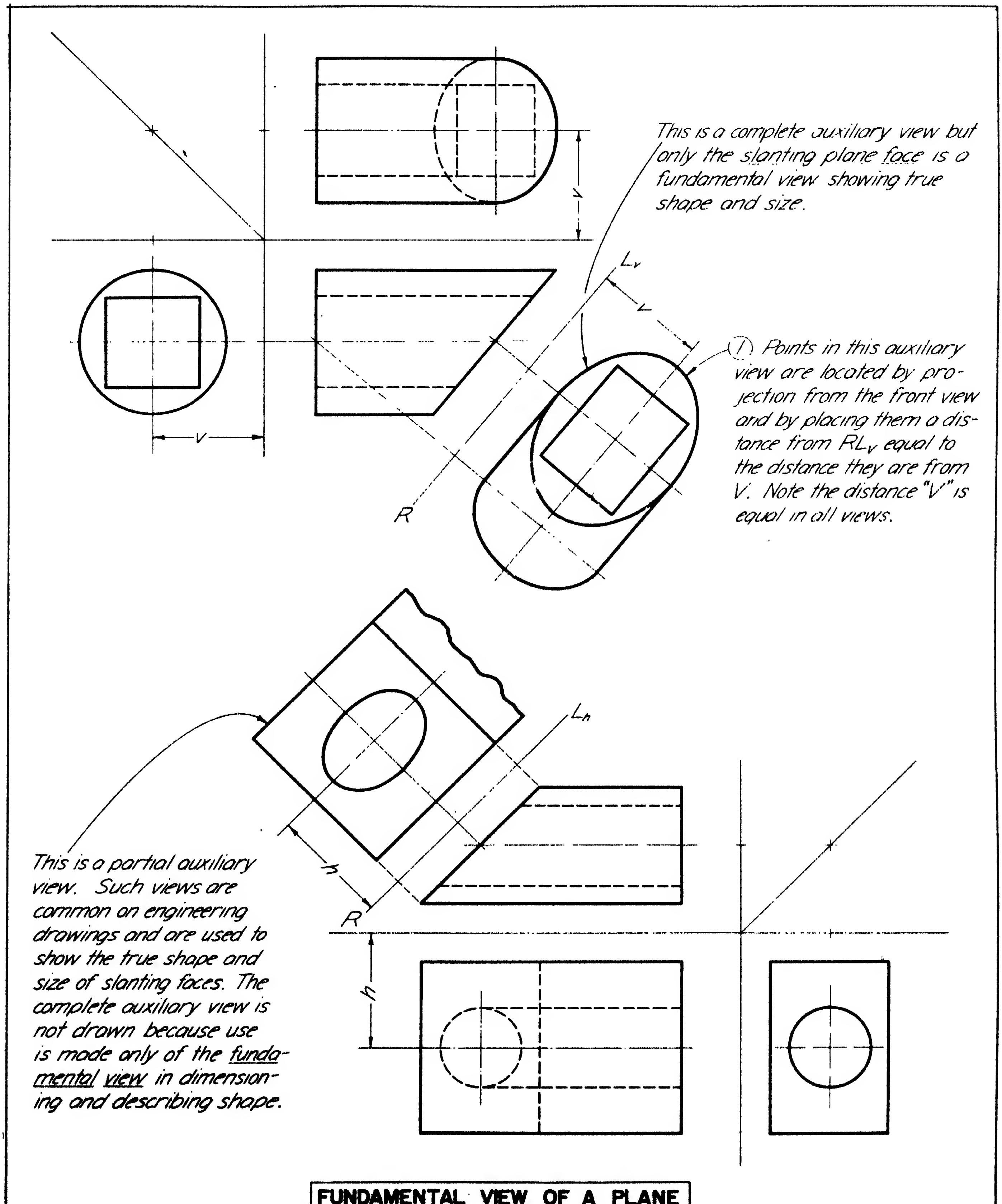


FIGURE 24

The derivation of a fundamental view of a plane.

36. **The Fundamental View of Oblique Planes.** Reference was made in Article 33 to the fact that to secure a fundamental view of an oblique plane an *edge view* of the plane must be available. When, as in Fig. 25, the plane is oblique it becomes necessary therefore *to find* an edge view. This is accomplished by the means of a *direction line.*

Direction line method. A direction line is any line contained in a plane which is parallel to one of the principal planes. One of its views, therefore, is a fundamental view, and an auxiliary plane may be located which will be perpendicular to the direction line by locating the edge view (RL) of the auxiliary plane at right angles to the fundamental view of the direction line. Since the auxiliary plane is perpendicular to the direction line, it will also be perpendicular to that plane in which the direction line was drawn. Therefore, the view of this plane of the direction line on the auxiliary plane perpendicular to it will be a line, or an edge view, of the plane.

This important principle is the means by which many perpendicular relationships are established as well as the device for solving problems involving oblique planes. A study of the drawings in Fig. 25 will assist in comprehending the method.

Top Drawing. Figure 25. An hexagon, with its circumscribing circle, is shown by a plan and elevation view. The problem is to draw the true shape and size of this figure. Since the plane of this figure is oblique to H, V, and P:

1. Locate in the plane of the hexagon a *direction line* AB. Since AB is drawn parallel to V, the front view therefore is a fundamental view and auxiliary plane RL_v may be located perpendicular to line AB by making its edge view RL_v perpendicular to the fundamental front view ab. The view of AB on this auxiliary plane will be a point (ab) because AB is perpendicular to the auxiliary plane and the view of the hexagon will be an edge view (a line) because the plane of the hexagon is also perpendicular to plane RL_v.

2. On plane RL_v locate the edge view of the hexagon and circle.

3. Locate RL parallel to this edge view and on this plane RL locate the fundamental view of the hexagon and circle.

Bottom Drawing. Figure 25. The top, front, and left end view of an oblique triangle are given. The problem is to find its true shape and size.

1. Locate a direction line $A1$ in the triangle parallel to H. (Such a line could be parallel to V or to P as well.) Perpendicular to this direction line locate auxiliary plane RL_h. Find the edge view of the triangle on plane RL_h. Parallel to this edge view now locate plane RL, and upon this second auxiliary plane derive the fundamental view of the triangle showing its true shape and size.

Note that a direction line may be placed parallel to H, or to V, or to P. The position of the auxiliary plane will, of course, be dependent upon this location, but the principle of finding the edge view is the same.

A horizontal direction line in the plane of an ore body is called the *strike*; the inclination of such a plane to the horizontal is called the *dip*. When the strike is stated as a bearing and the dip as an angle the position of such a plane may be located.

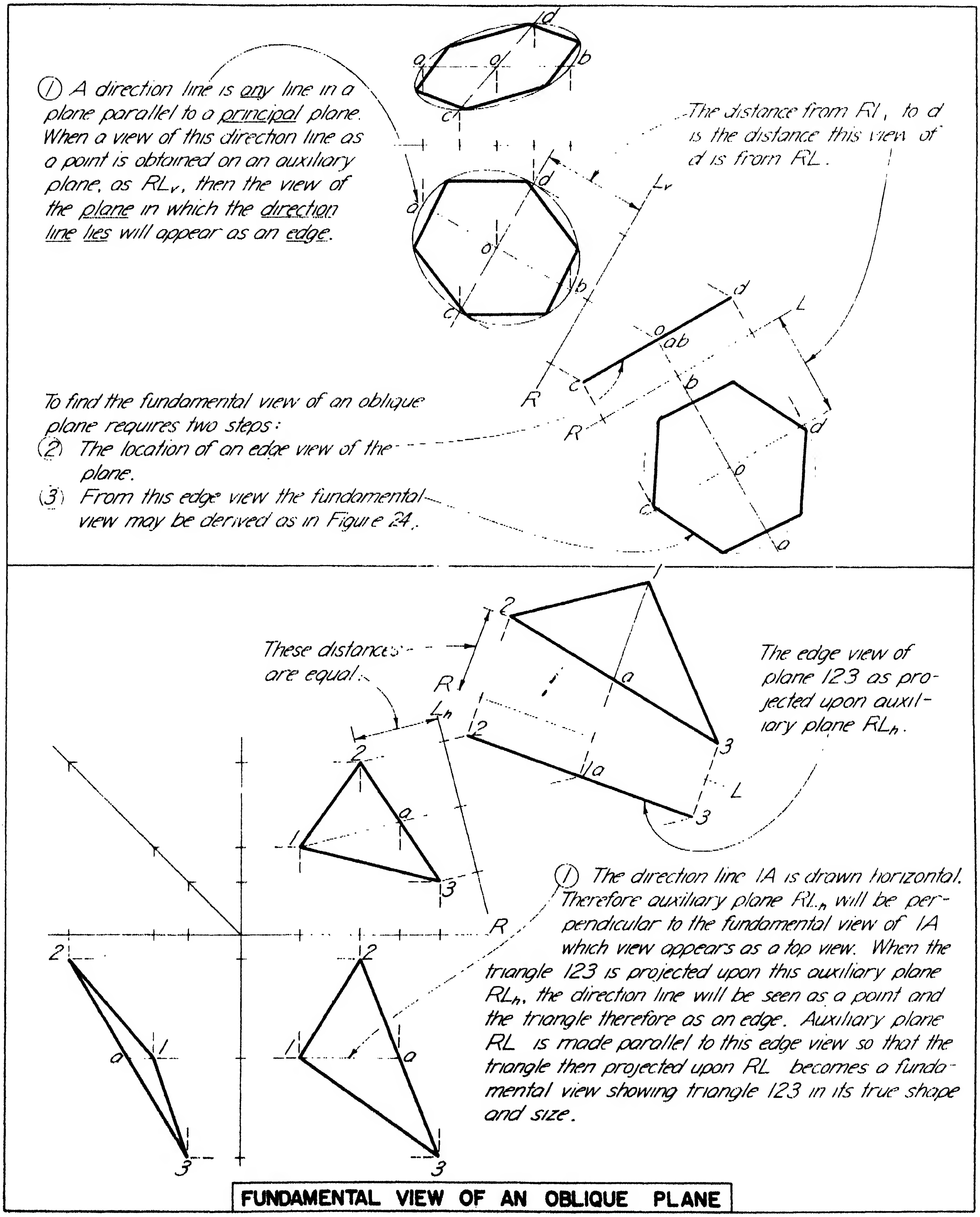


Figure 25

Fundamental views of oblique planes.

37. **EP4. To measure the angle between two lines.** An angle lies in a plane. The plane of the angle is fixed as to location by the two intersecting lines which form the angle. Therefore, the problem resolves itself into finding a fundamental view of the plane of these two intersecting lines, thus showing the true shape and size of the angle.

Top Drawing. Figure 26. An angle ABC is described by three views. The problem is to measure the size of this angle.

1. Draw a horizontal direction line AX in the plane of the angle. This is done by connecting A, one point in the plane of the angle, with X, any point on line BC of the angle, by a line which is parallel to H.

2. Locate auxiliary plane RL_h perpendicular to this direction line, and locate the edge view of angle ABC on this auxiliary reference plane. Note that RL_h is perpendicular to the *fundamental* view of the *direction line*.

3. Locate RL, a second auxiliary plane, parallel to the angle as shown by its edge view. Auxiliary plane RL must also be perpendicular to auxiliary plane RL_h. By projection and measurement, locate the view of angle ABC on plane RL, and measure the fundamental view of the angle as shown here in its true shape and size.

Bottom Drawing. Figure 26. The oblique line MN and a point A are given and shown by a top, front, and left end view. The problem is to construct these three principal views of an angle of given size from A to MN.

1. Find the edge view of the plane determined by line MN and point A. In this case the direction line AX was made parallel to V and, therefore, RL_v is perpendicular to the front view ax which is the fundamental view.

2. Locate RL parallel to the edge view of the plane of MN and A, and upon the plane RL project MN, which will be a fundamental view of MN, and point A. Draw ao in this fundamental view of plane MN and A, so that aon is equal in size to the given angle, and thus locate its vertex O.

3. By projection obtain the three required views of this vertex O and complete the three required views of angle AON.

It is to be noted that two solutions of this problem are possible since AO may slope in two directions when making the given angle with line MN.

It is also to be noted that if the given angle were 90° only one solution would be possible, and the method can be applied to erect a perpendicular from a given point to a given line.

By this same method, a line of stated length may be drawn from a given point to a given line, and in such a problem two solutions would be possible.

In studying the angular relations of lines it should be thoroughly understood that the true shape and size of the angle between two lines will be shown only on that plane of projection which is parallel to the two intersecting lines, or the three points, or the line and the point which determine the angle. Also that such a fundamental view may be obtained only when some one view, either a principal view or an auxiliary view, of the plane of the angle is shown as an edge view.

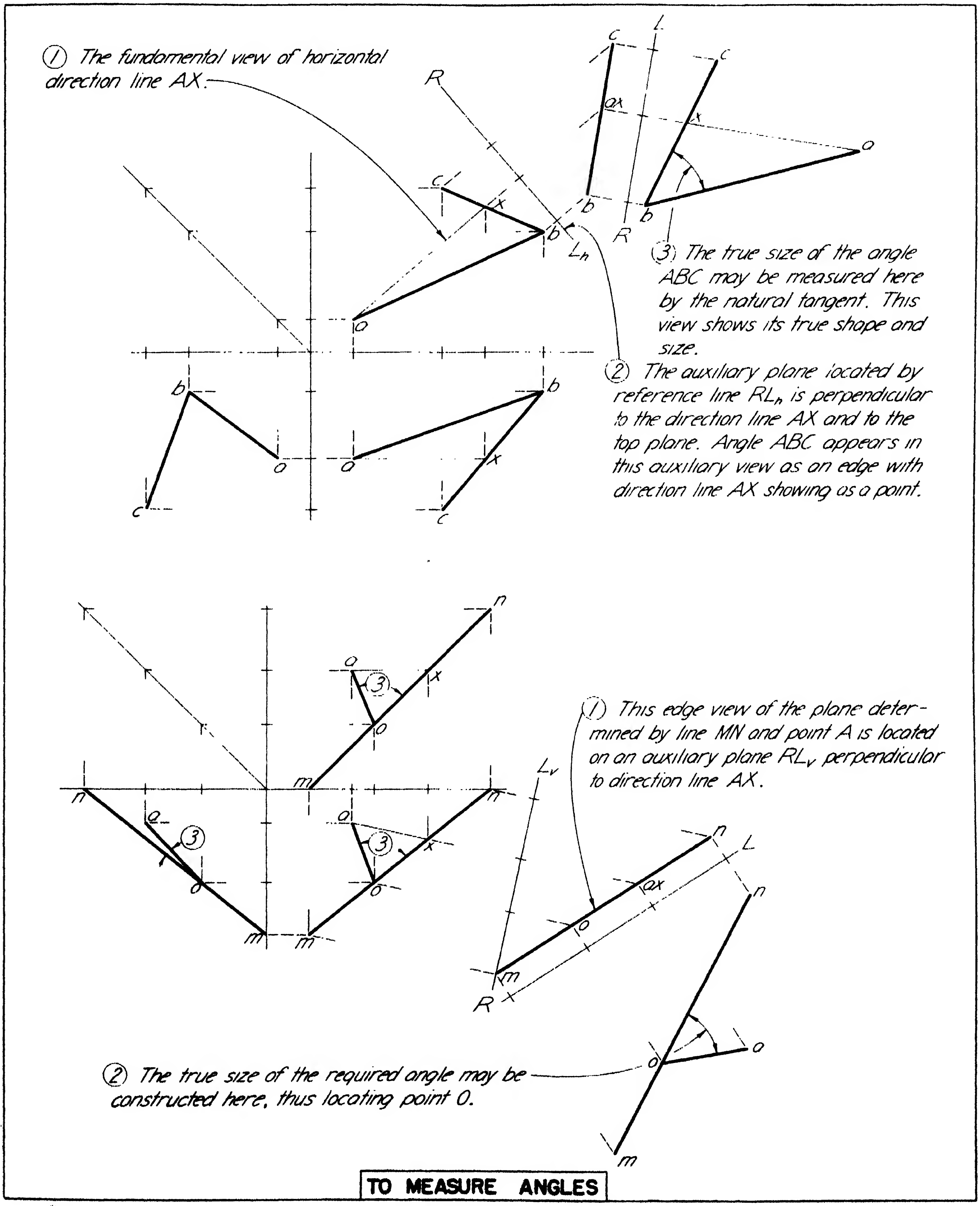


FIGURE 26
Essential Principle 4.

38. **To locate a line in a specified position.** In drawing-board work it frequently becomes necessary to locate a line in a specified position with respect to the principal planes, and with respect to other points or lines or planes of the problem.

If the specified position requires the line to be parallel or perpendicular to the principal planes and a stated distance from them, the relation of the three views of the line to the principal planes may be discovered by referring to Article 24, Figs. 16*a* and 16*b*.

If the line is to be located at a given slope, in a given direction, and to be of given (or indefinite) length, the method of locating such a line is explained in EP2, Article 31, Fig. 20.

If the line is to have specified angular relations to a second line, such a location may be obtained by the application of EP4, Article 37, Fig. 26, and Article 32, Fig. 21.

If, however, the line is to be located in a given oblique plane and is also to meet specified conditions with respect to its slope, or its direction, or its length, or combination of these, reference to Fig. 27 will be helpful in understanding the principles involved in locating the views of such a line.

In Fig. 27, three principal views of line *OP* are given. The problem is to locate lines which are perpendicular to *OP* and at the same time parallel to one of the principal planes. The lines are also to be of stated length (refer to Article 32) through point *O*.

To locate a line of given length perpendicular to *OP* and parallel to *H*.

1. Draw line 34 parallel to *H* by making the front view parallel to the reference line between *H* and *V*.

2. Draw the top view of 34 perpendicular to the top view of *OP*. Since this is a fundamental view it will make 90° with the top view of *OP* and will be the given length. By projection, from the top view of 3 and 4 locate the front view of 3 and 4.

To locate a line of given length perpendicular to *OP* and parallel to *V*.

3. Draw line 12 parallel to *V*; make its top view parallel to the reference line between *H* and *V*, and its front (fundamental) view perpendicular to the front view of *OP* and of the given length.

To locate a line of given length perpendicular to *OP* and parallel to *P*.

4. Draw the fundamental end view of 56 perpendicular to the end view of *OP* and of the given length, and locate the front and top views by projection. The front view is parallel to the reference line between *V* and *P*, and the top view is located directly above it by measurement and projection.

Since these lines are all perpendicular to *OP*, they all lie in a plane perpendicular to *OP*. Since they are all of equal length, points 1-2-3-4-5-6 lie on the circumference of a circle whose several views are shown in dashed lines, and whose plane is perpendicular to line *OP*.

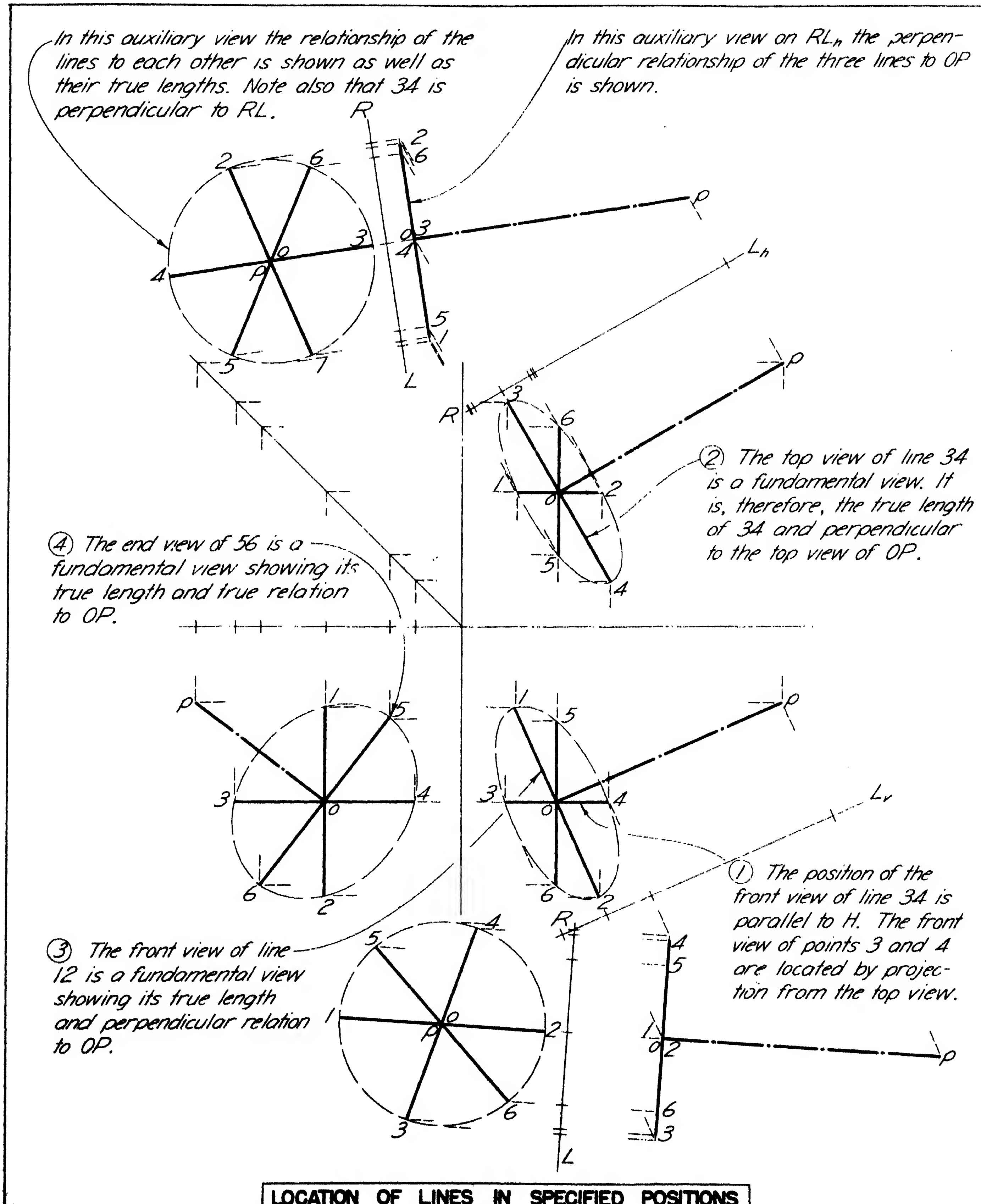


FIGURE 27
The location of lines in specified positions.

39. **EP5. To construct a plane figure of given shape, size, and location.**

Top Drawing. Figure 28. Two lines, *MN* and *AB*, are given by their top and front views. These lines intersect at *O* and locate the plane of a regular hexagon which is 2 inches across the corners. The problem is to locate the top and front views of such a hexagon.

1. Locate the edge view of the plane of *MN* and *AB* by connecting any two points of these lines (as *A* and *X*) with a direction line *AX* parallel to *V*. This edge view appears on plane RL_r, which plane is perpendicular to direction line *AX*.

2. Find the fundamental views of lines *MN* and *AX* on auxiliary plane *RL* which is parallel to the edge view. About the fundamental view of *O*, which is the center of the hexagon, construct the fundamental view of a regular hexagon inscribed in a 2-inch circle using 2 inches of *MN* as one diameter.

By projection and measurement, the corners of the hexagon may be located by front and top views; when these corners are joined in the correct order, the top and front views of the required hexagon will be found.

Since no specified position for a long diameter of the hexagon was given, an indefinite number of solutions is possible. The solution chosen assumed such a diameter to coincide with line *MN*. If, however, a specified position of a long diameter were given, as for example parallel to *V*, or to *H*, etc., the position of such a diameter would have to be located as described in Article 38, Fig. 27.

Bottom Drawing. Figure 28. The base line *AB* and the center line *MN* of a lug are shown in plan and elevation. The problem is to construct the top and front views of the lug which is described as to shape and size in the small accompanying drawing.

1. Locate the fundamental views of the base *AB* and the center line *MN*, on auxiliary plane *RL*. In this fundamental view draw the true shape and size of the lug, using these lines as base and center lines.

2. In locating the top and front views by projection and measurement, observe that a series of horizontal lines parallel to *AB* may be used conveniently. The line 12 is drawn first, for example, in the fundamental view, and by projection and measurement points 1 and 2 are transferred to the top and front views. By drawing a number of such lines enough points may be located to define completely the top and front views of the lug. Note that *horizontal* lines as shown in the fundamental view are perpendicular to *RL*.

This problem occurs not infrequently in drawing-board work when lugs or "ears" are to be located in positions oblique to the principal planes, or when structural plates connecting an inclined member are to be designed.

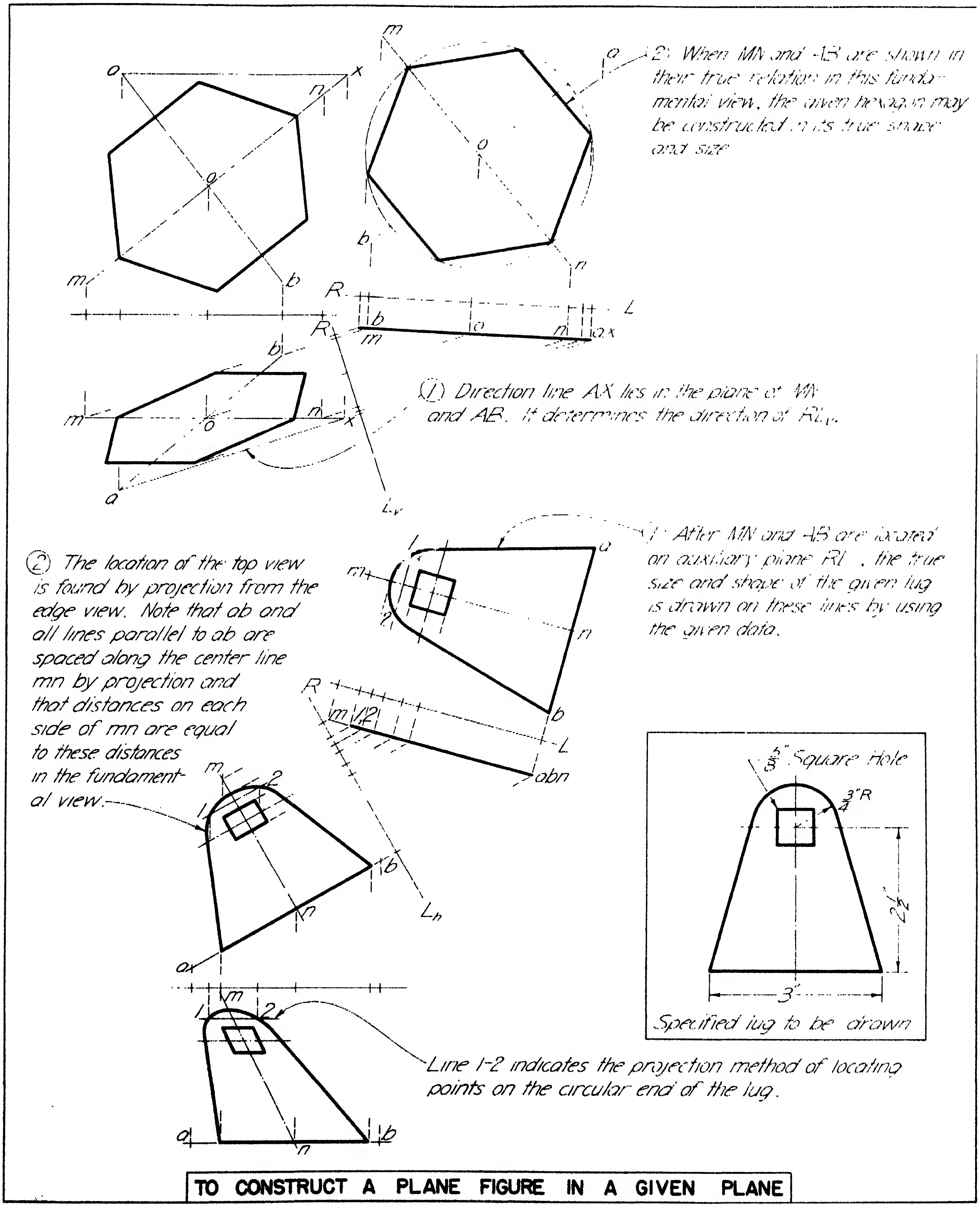


Figure 28
Essential Principle 5.

40. **EP6. To find where a line pierces a plane.** The given plane, it is to be observed, may be located by three points, a line and a point, two parallel lines, two intersecting lines, or by its boundaries, either straight or curved lines. In any and all such cases the piercing point may be located; and if it becomes desirable to define a plane given by intersecting lines, or parallel lines, or by three points, or by a line and a point, by boundary lines this may be done by arbitrarily assuming boundaries by connecting two intersecting lines by a third, or by connecting two parallel lines by two other parallel lines, or by joining three points or by joining two points of the line with the given point.

Top Drawing. Figure 29. The plane is given as a triangle 123 (three given points) and is shown by the top and front views of the triangle. The given line is AB. The problem is to find where (if, indeed, at any point) the line AB pierces the plane 123.

1. Locate the edge view of triangle 123 on auxiliary plane RL_h, and on this same plane locate the auxiliary view of AB. Note that a horizontal direction line $2X$ in the plane of the triangle fixes the direction of RL_h.

2. The point where AB enters the plane of 123 is located where the auxiliary view of AB on plane RL_h intersects the edge view of the triangle. This point O may now be located on AB in the top and front views by projection.

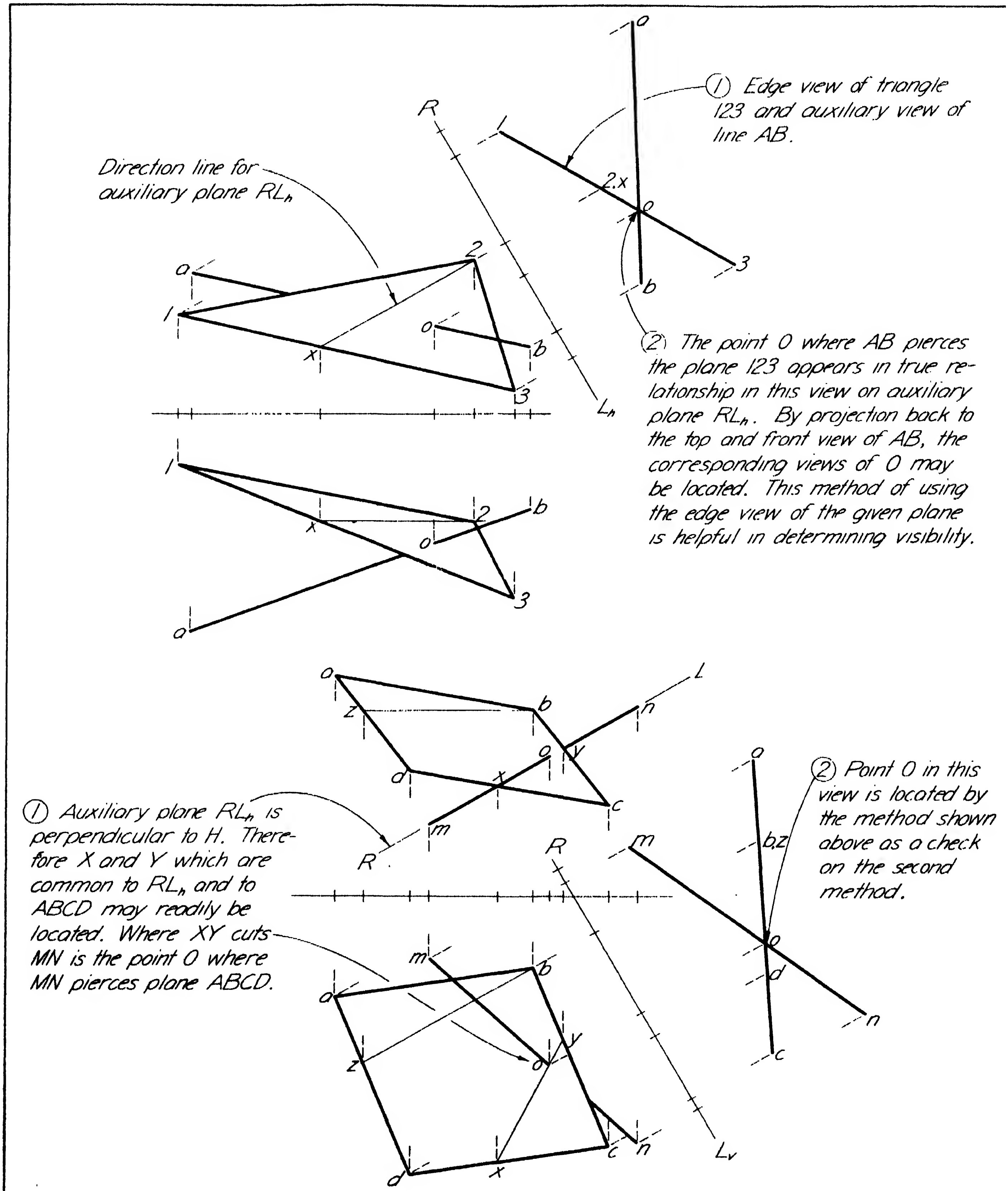
It is to be observed that, if the auxiliary view ab does not intersect the edge view 123, then AB does not pierce 123; it is also to be observed that, if the auxiliary view ab does not intersect the edge view between points 1 and 3, then AB does not intersect the plane of the triangle 123 *within the given area.*

Bottom Drawing. Figure 29. A second method of locating the piercing point is by means of an auxiliary plane RL_h perpendicular to H and containing the given line. The plane is located by the top and front views of the boundaries $ABCD$. The given line is MN. To find where MN pierces the plane $ABCD$:

1. Locate RL_h, the auxiliary plane perpendicular to H, which also contains line MN. Locate the line XY (by top and front views) which is common to plane RL_h and plane $ABCD$. This line XY intersects the given line MN at O. The point O, therefore, is the piercing point since O is on line MN and also on line XY which is in plane $ABCD$.

2. The piercing point is also found on plane RL_v as in the top drawing, both as a check on this second method, and also to illustrate that the edge-view method makes visibility more easily determined. An examination of the problem shows how portions of the line are hidden by the plane, and how this is made clear in the edge view.

It is to be noted that, so long as the auxiliary view of MN intersects the edge view of the plane $ABCD$, the line MN will pierce this plane but *not necessarily within the limits set by the boundary lines.* Since the piercing point is on line MN its projections may readily be found and identified. If these projections fall outside the given views of the plane, such a situation simply means that, while the line pierces the plane, the piercing point is beyond the given boundaries of the plane.



POINT WHERE A LINE PIERCES A PLANE

Figure 29
Essential Principle 6.

41. **EP7. The Principle of Revolving.** In the study and solution of problems by graphic methods, the method of *changing* the *position* of the *given data* into a fundamental relation to the principal planes of projection is a convenient and useful graphic manipulation. This is known as the principle of revolving.

Top Drawing. Figure 30. *A line may be revolved* about any of its points into a new position parallel to a projection plane. In this new position, the line will appear as a fundamental view. In this drawing AB is an oblique line shown by three views. It is to be revolved about A into a new position parallel to V, the front plane. It is important to note that during this movement the point B describes a circle *whose plane is parallel to H.* In other words, AB is so revolved about A that the original relation of AB to the top plane is *not changed* and *only its relation* to V is altered. When AB, therefore, becomes parallel to V its plan view is easily found and from this view the new front (fundamental) view and the new end view can be derived by projection.

Middle Drawing. Figure 30. *A plane may be revolved* about an appropriate direction line into a new position parallel to a projection plane. The direction line must itself be parallel to the projection plane parallel to which the given plane is to be moved. In the triangle 123, the side 12 is already parallel to V and, therefore, may be used as an axial line of rotation. (If such were not the case, a direction line in plane 123 would have to be drawn parallel to the plane to which the triangle is to be made parallel.) About this line 12 as an axial line, triangle 123 may be revolved parallel to V. (The triangle could not be revolved parallel to any other plane about this line.) The line 12 simply rotates on its own axis, and the new location of point 3 must be found. The method of doing this is by use of an auxiliary plane RL_r perpendicular to axial line 12, on which the path of rotation for point 3 will appear as a circle. When point 3 in this view arrives at a point where the drawing shows it to be the same distance from RL_r as are points 1 and 2, then the triangle is parallel to V. The front view of this path of rotation is an edge view of the circle; therefore the front view of 3 may be located by projection from the auxiliary view 3, and the top view of 3 may be found by projection from the front view and measurement from the auxiliary view. The two views 123 show the triangle in its new position parallel to V.

Bottom Drawing. Figure 30. *An object may be revolved* about an axis into a fundamental position parallel to some one plane of projection. An oblique cylinder is shown by partial top and front views in solid lines in its original position. To revolve this cylinder parallel to H, an axis OP perpendicular to V and parallel to H is used. The dashed lines show the cylinder in its new position parallel to H. See also Fig. 9*b*.

The basic principle to be observed in revolving is to choose an axis and control the path of rotation so that *one view* of the *object in its new position* can readily *be drawn.* From this one view the other views may be derived by projection when the character of the motion from the first position of the object to its second position is controlled and understood.

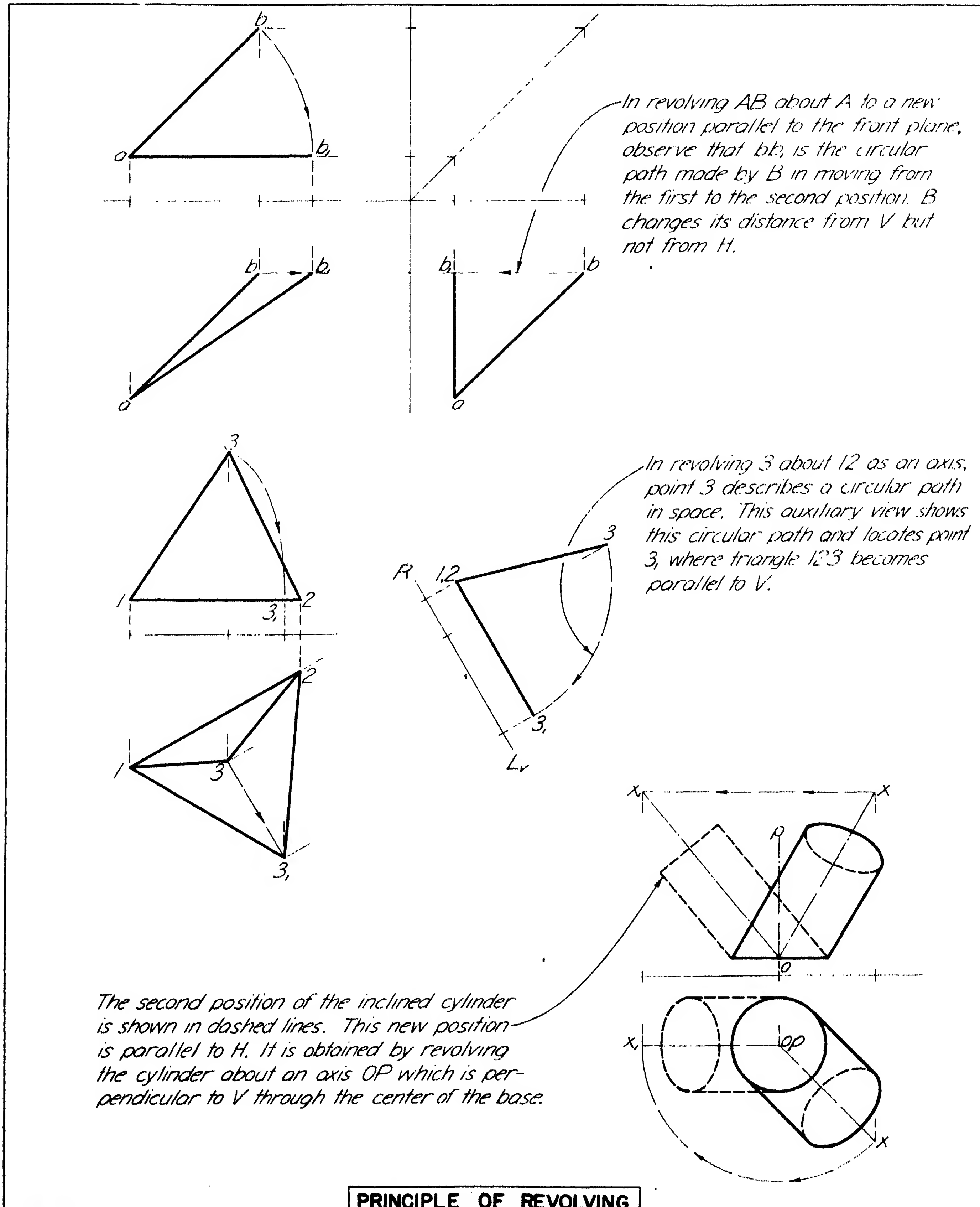


FIGURE 30
Essential Principle 7.

42. **Perpendicular Relationships of Lines to Planes.** In Article 32, Fig. 21, the perpendicular relationships between mutually perpendicular lines were explained and the method of securing the necessary views to describe them was illustrated. This same principle is used in establishing the relationship of lines perpendicular to planes.

A line is perpendicular to a plane when the plane contains lines whose fundamental views are perpendicular to corresponding projections of the given line.

To locate a plane perpendicular to a given line. Top Drawing, Fig. 31. An oblique line *OP* is located by its plan and elevation. To locate the position of a plane through point *O* perpendicular to *OP*:

1. Draw the top (fundamental) view of a line 12 perpendicular to the plan view *OP*. The front view of 12 must be horizontal.

2. In similar fashion locate line 34 parallel to *V* so that its front (fundamental) view will be perpendicular to the front view of line *OP*.

Any plane containing 12 and 34 will be perpendicular to line *OP* and, since 12 and 34 intersect a plane, is located by them.

To locate a line perpendicular to a given plane. Middle Drawing, Fig. 31. An oblique plane 1234 is located by its boundary lines 1234. To locate the position of a line perpendicular to this plane at any point *O*:

1. Draw the front and the top (fundamental) view of a horizontal line *AX* through point *O*. This line *AX* must lie in plane 1234. The top view of the required line *OP* may now be drawn perpendicular to the top (fundamental) view of *AX*. *OP* may be of any assumed length.

2. In similar fashion, a direction line *BY* may be drawn through point *O* parallel to the vertical plane. Since the front view of *BY* is a fundamental view, the front view of the required line may be drawn perpendicular to *BY* at *O*.

To locate a plane figure of stated shape and size perpendicular to a given line and and in a stated position. Bottom Drawing, Fig. 31. An oblique line *OP* is located by its top and front views. It is required to draw a 3-inch square perpendicular to *OP* at *O* so that two edges of the square will be horizontal lines.

1. Draw a horizontal direction line *AX* through *O* by making the top view of *AX* perpendicular to the top view of *OP*. Make *AX* 3 inches long.

2. Draw a fundamental view of *OP* on plane RL_A and locate the edge view of a 3-inch square perpendicular to *OP*. The specified horizontal position and length of two edges may be shown on *RL* as a fundamental view, although this fundamental view is not needed for the location.

3. The top view of the 3-inch square may be drawn by projection from the edge view. Since 12 and 34 are to be horizontal lines their top views will be 3 inches long and parallel to *AX* since these views are fundamental views.

4. The front view of the square may be obtained by projection from the top view since the length and position of lines 12 and 34 are known.

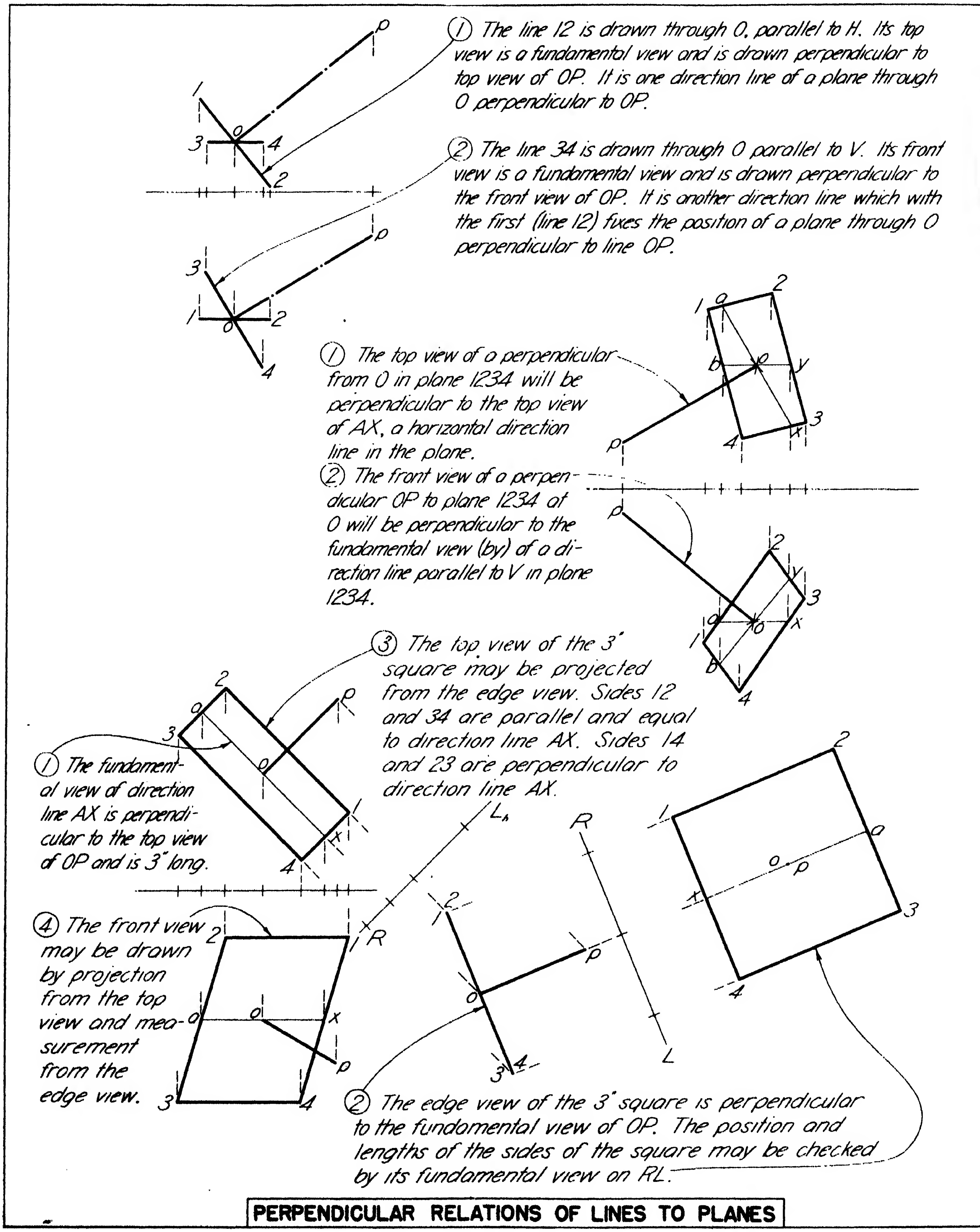


Figure 31

Perpendicular relationships between planes and lines.

43. EP8. To locate and measure a perpendicular from a point to a plane.

In addition to measuring the shortest distance, the problem requires this perpendicular to be *located* in the top and front views.

Top Drawing. Figure 32. A plane 1234 in the form of a quadrilateral is shown by its top and front views. The problem is to *locate* and *measure* the shortest line—a perpendicular—from P to plane 1234. To find and measure the shortest distance:

1. Locate an edge view of 1234 and an auxiliary view of P on plane RL_r which is perpendicular to direction line $2X$. From this view of P, the auxiliary view of PO is now drawn perpendicular to the edge view of the plane. This auxiliary view of PO may be measured for length, since it is a fundamental view.

2. To locate the front and top views of PO, draw the front view po perpendicular to the front view of direction line $2X$. Point O may now be located on the front view by projection and on the top view by projection from the front and measurement from the view on RL_r.

Bottom Drawing. Figure 32. In this figure three views of a hexagon are given. The problem is to locate P—the apex of a *right* pyramid of which this hexagon is the base—a given distance from O.

1. Draw top and front views of OZ perpendicular to the plane of the hexagon by making these views perpendicular to the appropriate direction lines of the hexagon, as 63 and xy.

2. Find the true length of OZ by revolving OZ parallel to V. (See Article 41, Fig. 30.) On oz_1, this true length, lay off op_1, the given length of the altitude of the right pyramid. Revolve oz_1 back to its original position, and find the front and top views of P.

These views of P are the location of the apex of the pyramid the given distance from O.

The top, front, and end views of the pyramid may be drawn by joining apex P with each of the six corners of the hexagonal base. To avoid a confusion of lines in this drawing only the end view is so shown. This view assumes the pyramid to be a solid. If such were not the case, edges $3P$, $4P$, and $5P$ would be visible up to the point where they disappear behind the edge 16 of the base.

In the study of text drawings where planes are defined by outlines, students will find it helpful to shade the surface which is visible. A convenient way of doing this is to use the flattened face of a colored pencil and lightly shade the area. Select blue, for example, as the color for the face visible when viewed through the top plane and shade in blue *every view* of this face where visible. In like fashion red may be chosen to identify plane areas visible through the front plane, and *all views* of such a face then should be shaded red if this face is *visible* in the view so shaded.

Such shading tends to assist materially in the visualization of the relationship since the visibility of the two faces of a plane area is differentiated.

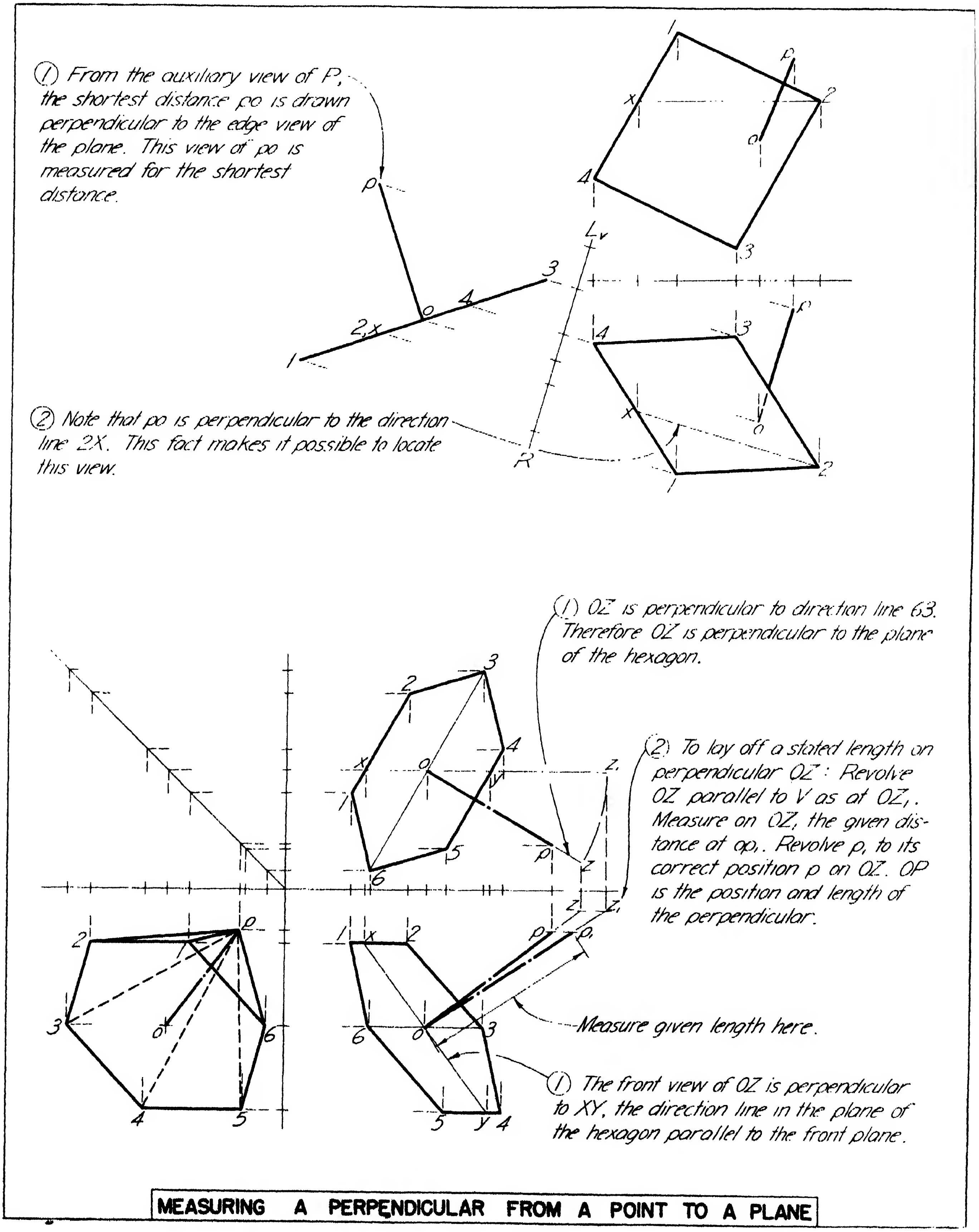


FIGURE 32
Essential Principle 8.

44. EP9. To locate and measure the shortest line connecting two lines which are neither parallel nor intersecting.

Top Drawing. Figure 33. The top and front views of AB and MN are given. The problem is to locate and measure XY, the shortest connecting line.

1. Locate a fundamental view of MN on RL_h and, on this same plane, an auxiliary view of AB.

2. Locate a point view of MN on auxiliary plane RL and on this same plane RL an auxiliary view of AB. Since MN appears as a point on plane RL, xy may be drawn perpendicular from mn to ab in this view and xy will be a fundamental view of the shortest connecting (perpendicular) line showing its true length.

3. The front and top view of XY may be obtained by projection from the auxiliary views.

Bottom Drawing. Figure 33. The above problem is solved by a second method in this drawing.

1. Line AB is placed in a plane parallel to MN. This plane is located by drawing, through any point O on line AB, a secondary line M_1N_1 parallel to the given line MN.

2. Draw the direction line AZ in this secondary plane and locate RL_v perpendicular to this plane containing AB and M_1N_1, the parallel to MN. Find the auxiliary views of both given lines AB and MN, and note that they are parallel in this view.

3. Any perpendicular between these two auxiliary views will be a measure of the shortest distance, but, to fix the actual position of the shortest perpendicular line, the fundamental views of AB and MN on plane RL, which is parallel to AB and MN, must be located.

4. Their apparent intersection in this view determines the position of XY, which now may be transferred to plane RL_v and thence to the front and top views by projection.

The principle of the shortest connection has several applications. For example: the shortest *horizontal*, or the shortest *vertical*, or the shortest line of *given slope* may be required.

The shortest horizontal between two lines may be located by finding their *apparent* intersection in the front view; the shortest vertical between two lines may be located by finding their *apparent* intersection in the top view; the shortest line of given slope connecting two lines may be located by finding their *apparent* intersection on an auxiliary plane perpendicular to the line of given slope.

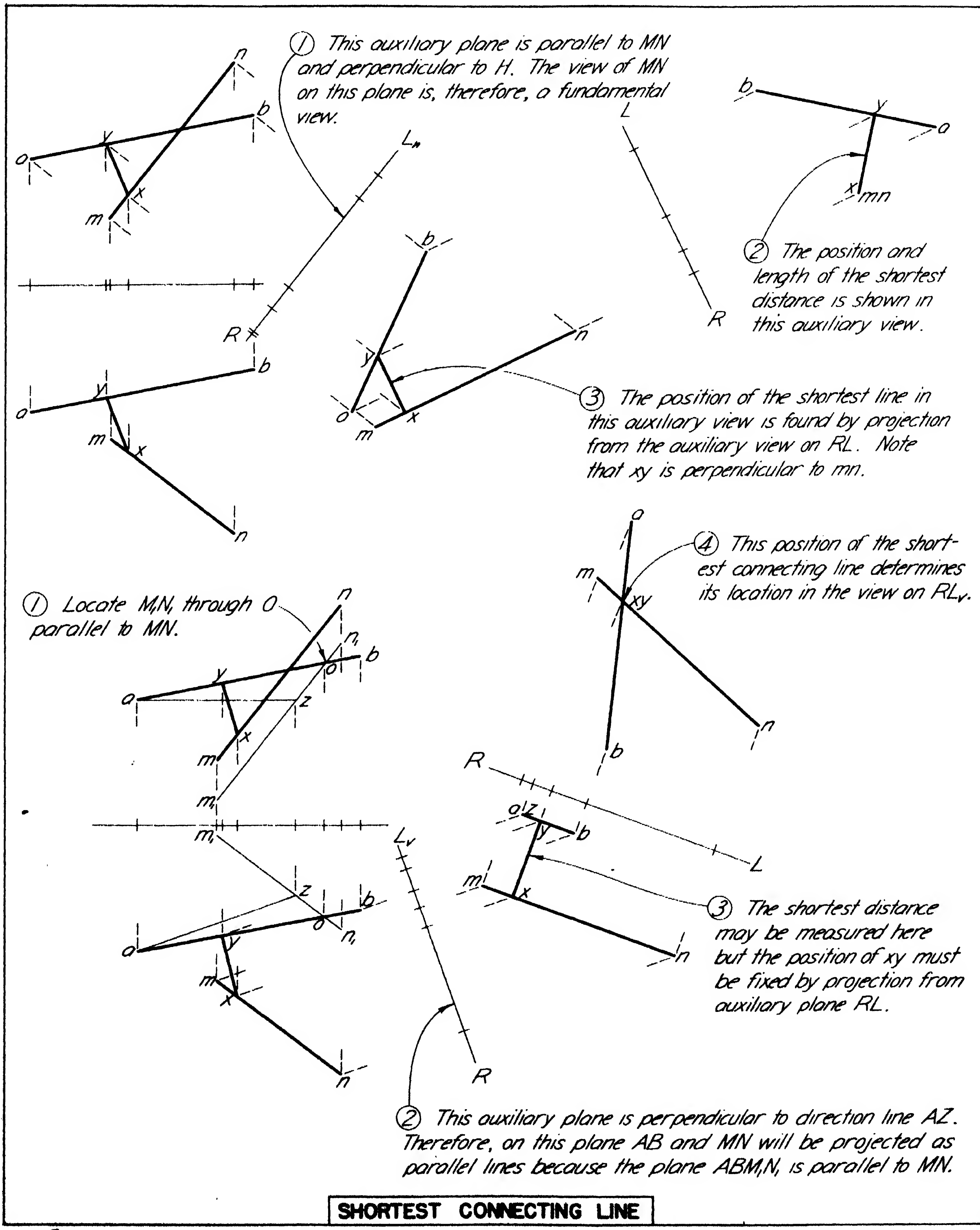


Figure 33

Essential Principle 9.

45. EP10. To measure the angle a line makes with a plane.

Top Drawing. Figure 34. The plane is given as the bottom face (1234) of a plinth. The line *PO* pierces this plane at a given point *O*. To find the angle *PO* makes with plane 1234:

1. Find the edge view of the plane and an auxiliary view of *OP* on plane RL_v which is perpendicular to edge 14 of the plane. (This edge 14 is a direction line because it is parallel to *V*.)

2. Find the true length of *PO* by revolving it parallel to *V*. (Article 41, Fig. 30.)

3. Lay off *po*, in the auxiliary view, equal to this true length, and *keep point P the same distance from plane 1234*. (So long as point *P* remains a constant distance from plane 1234 and from point *O*, the angular value does not change during rotation. When *PO* appears in its *true length,* the value of the angle appears in *its true size.*)

The angle thus formed in the auxiliary view between *po* and the edge view of the plane is the true size of the angle *PO* makes with plane 1234.

Bottom Drawing. Figure 34. The same problem may be solved by a second, and indirect, method.

1. Draw *PX* perpendicular to plane 1234 by making its front and top views perpendicular to corresponding direction lines. (The front view *px* is perpendicular to the front view of edge 14, a direction parallel to *V*; the top view *px* is perpendicular to any horizontal direction line of plane 1234, such as *BY*.) Since *PX* is *perpendicular* to plane 1234, angle *OPX* will be the *complement* of the angle *PO* makes with this plane.

2. *Draw a horizontal direction line* OX *in the plane of angle* OPX. Locate HL_h perpendicular to this direction line, and find the edge view, *opx*, of this angle on plane RL_h. Revolve the angle *OPX* about *ox* into a position parallel to the horizontal plane (note in the auxiliary view that this position is parallel to the reference line RL_h which is in *H*), and thus locate P_1. By projection from the revolved view oxp_1 locate the top view of this revolved position.

3. Since this top view of the revolved position is also a fundamental view (the revolved position being parallel to *H*), the true size of the complement of the angle will be shown. Now by graphical subtraction from 90° the true value of the angle *PO* makes with plane 1234 may be found and its value measured by the method of natural tangents. (See Article 29, Fig. 18.)

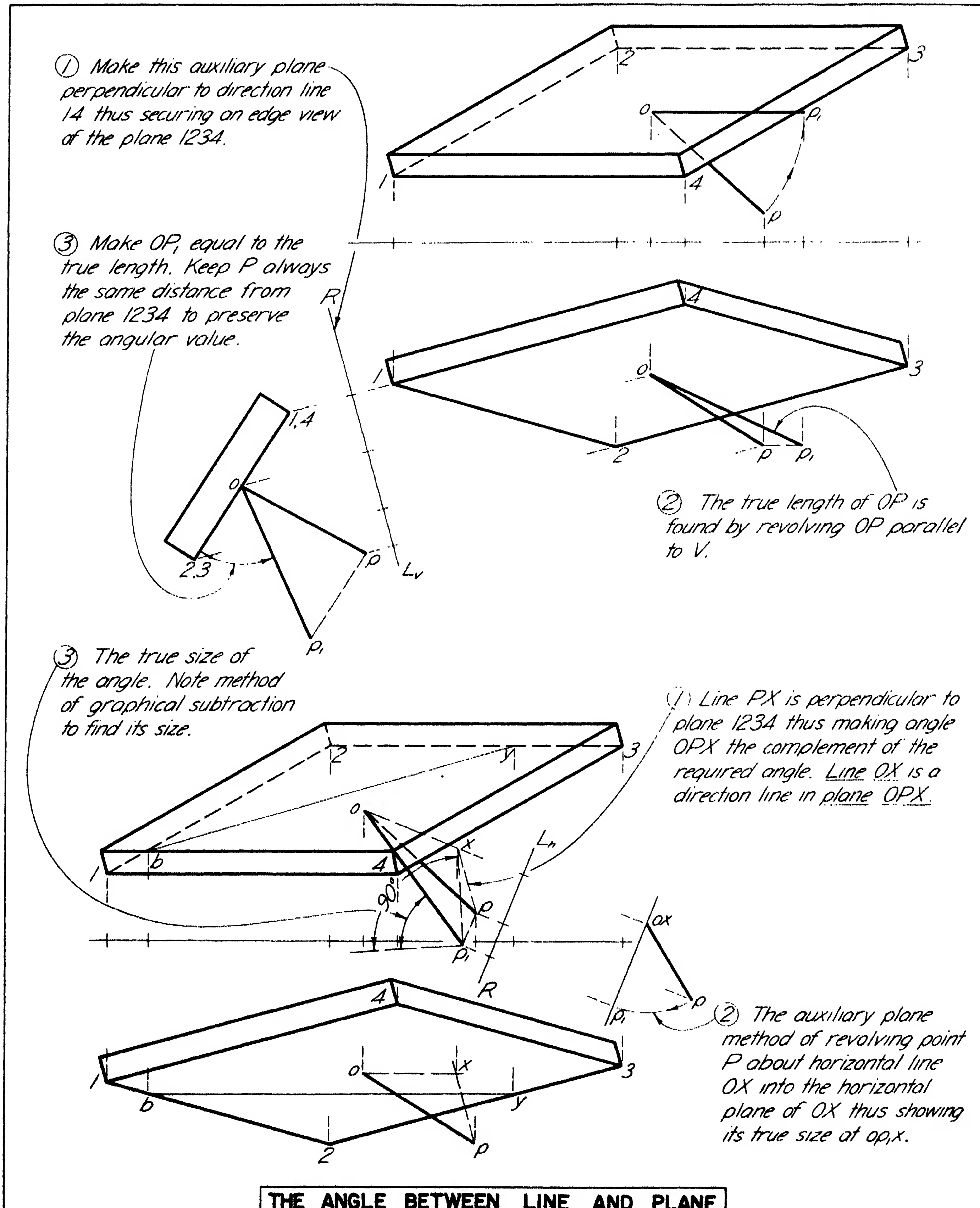


FIGURE 34

Essential Principle 10.

46. EP11. To locate the line of intersection between two planes.

Top Drawing. Figure 35. Two planes, a quadrilateral 1234, and a triangle *ABC* are given. The problem is to locate *OP*, their line of intersection. To find *OP*:

1. Draw direction line *CX* and locate RL_v perpendicular to plane *ABC*. Find the edge view of plane *ABC* upon auxiliary plane RL_v and also the auxiliary view of quadrilateral 1234. In this view the points where the lines of 1234 pierce the plane of *ABC* appear, and *OP*, the intersection, may be located.

By projection from RL_v, this intersection may be transferred to front and top view. (In this problem, *O* and *P* fall inside the boundaries of *ABC*. These planes might be shaped, however, so that *O*, for example, would fall outside. In such a case, line *AC* of the triangle would pierce inside the area 1234.)

Bottom Drawing. Figure 35. One plane—1234—is shaped as a quadrilateral; the other plane is located by two intersecting lines *CD* and *AB*. To find where the plane of these two lines intersects the plane 1234:

1. Locate the edge view of quadrilateral plane 1234 on plane RL_v which is perpendicular to direction line 3*X* of plane 1234; locate also the auxiliary views of *AB* and of *CD* on this plane RL_v. Find the points *P* and *O* where *AB* and *CD* pierce plane 1234 as shown in the auxiliary view. See Article 40, Fig. 29. Transfer, by projection, to front and top views, the points *P* and *O* on lines *AB* and *CD*. Draw the line *OP*, which is the line of intersection. (There may be cases where the area of 1234 will have to be extended beyond given boundaries in order to find where the given lines pierce.)

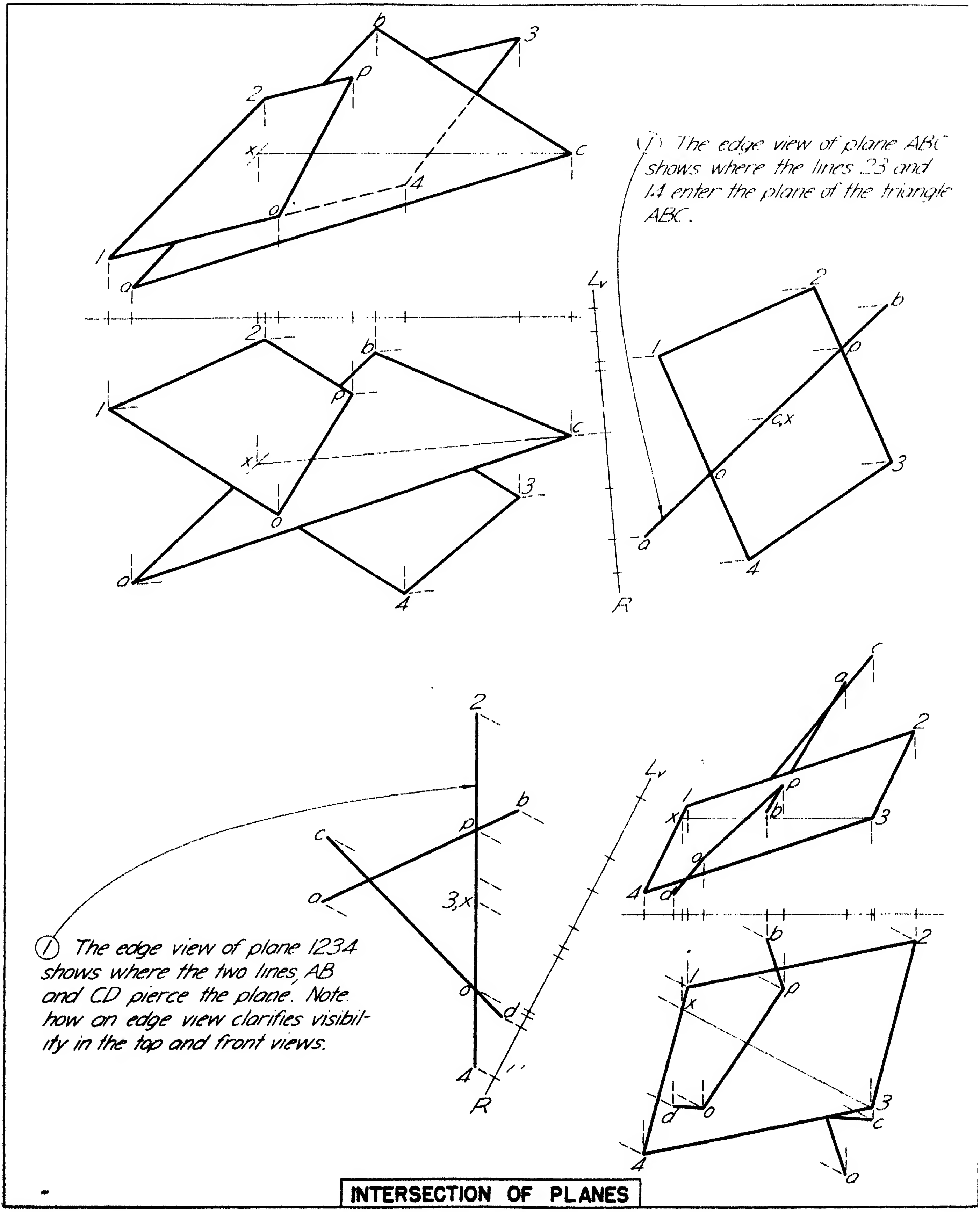


Figure 35

Essential Principle 11.

47. EP12. To measure the angle between two planes.

Top Drawing. Figure 36. A truncated pyramid, forming a hopper, is given. The problem is to measure the dihedral angle between planes $1AB2$ and $2BC3$. Since the angle between two planes lies in a plane perpendicular to their line of intersection, this angle will be shown in its true shape and size in the view where the common line is projected as a point. To find the angle:

1. Locate a fundamental view of $2B$, the common line on which must lie the vertex of the dihedral angle, on auxiliary plane RL_r. On this same auxiliary plane, locate planes $1AB2$ and $2BC3$. (The entire pyramid is shown in the drawing.) Locate RL perpendicular to the fundamental view of $2B$ and find the view of $B2$ on this plane (it will appear as a point) and of 23 and of 21. Measure the angle $3B1$, which will be the true size of the dihedral angle required.

It will be observed that the size of the angle between the planes is measured in the auxiliary view by the diveregnce of lines 12 and 23. In problem work, therefore, an auxiliary view of the whole pyramid is not needed.

Bottom Drawing. Figure 36. These same methods may be used to locate a plane perpendicular to a given plane along a given common line of intersection. Quadrilateral 1234 is given and a plane is to be found which will be perpendicular to 1234 along line PO.

1. By double auxiliary projection locate the point view op and the auxiliary view of $O2$. Since the required plane is to make 90° with 1234, draw oz on plane RL perpendicular to $O2$.

2. By projection and measurement locate Z in the front and top views. To give the plane shape and appearance, boundary lines parallel to OP and OZ may be drawn, thus representing a plane perpendicular to 1234 at the line OP.

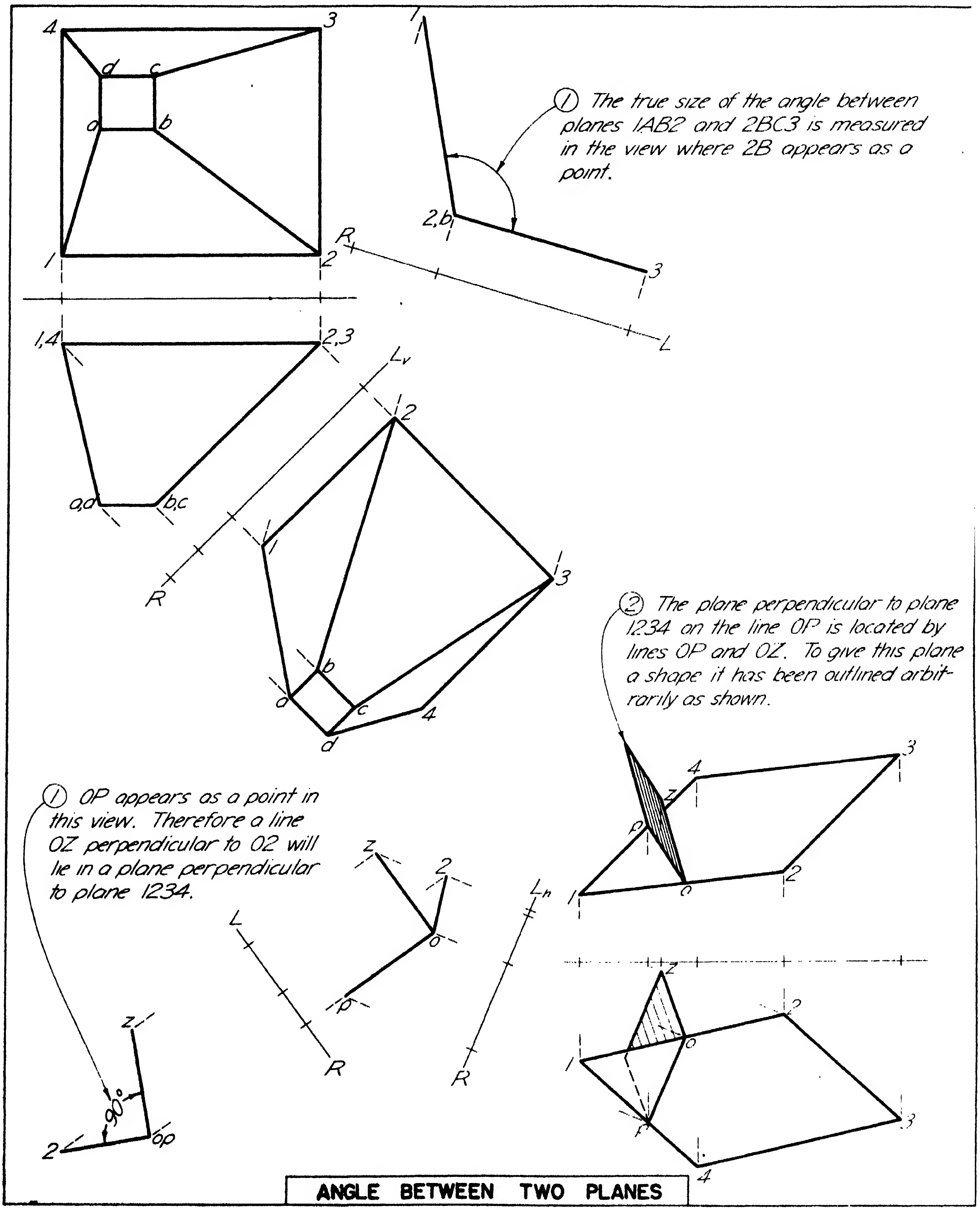
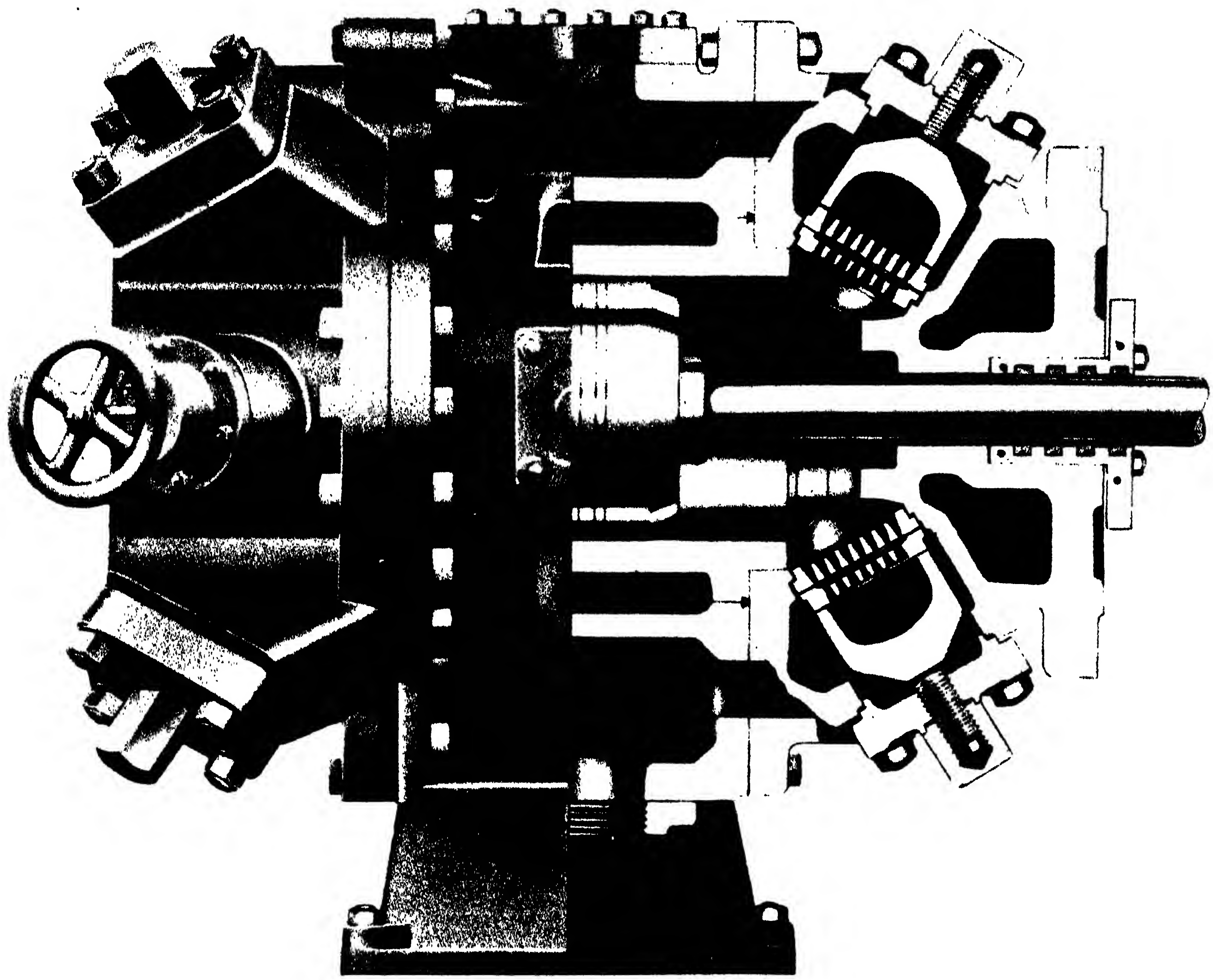


FIGURE 36
Essential Principle 12.



The design and construction of machinery require many problems to be worked out on the drawing board. In addition to representation in elevation and section, many shapes on this compression cylinder require auxiliary views and the intersection of surfaces for their design.

CHAPTER V

SURFACES

48. A surface is an area—either plane or curved—made by a line—either straight or curved—whose successive positions in space are established by a known relationship. For example: a straight line may move so as constantly to remain parallel and touching a plane curve, thus generating a cylindrical surface; a circle may revolve about one of its diameters, thus generating a sphere; a straight line may move so as constantly to touch a circle and a straight line and be parallel to a given plane, thus generating a warped surface.

In drafting work a surface may appear as a solid or with the volume inside the area as a void (as a hollow cylinder). The problem which first concerns the draftsman is how to represent surfaces on a drawing and how to identify the nature of the surface to be represented. In the pages which follow is given a brief discussion of the classification, the identification, and the representation of surfaces, together with the solution of the more common and essential problems arising in drawing-board work relating to surfaces.

49. In general, surfaces are either *ruled surfaces*, or *double-curved surfaces*. A ruled surface is one on which can be drawn straight lines, or elements. A double-curved surface is one which curves in *two* directions and on the surface of which no straight lines may be drawn.

Surfaces are, therefore, classified as (1) plane, single-curved, and warped, all of which are ruled surfaces; and (2) double-curved surfaces, most of which are surfaces of revolution, and none of which are ruled.

50. To identify the nature of a surface the straight-edge method is convenient. If a straight edge can be made to coincide with a surface, it is *ruled*. If a straight edge cannot be made to coincide with a surface, it is *double curved*.

If a surface is ruled, it will be either (1) a plane surface, in which case the straight edge will coincide in *any* direction; (2) a cylindrical surface, in which case successive positions of the straight edge will be parallel; (3) a conical surface, in which case successive positions of the straight edge will pass through one point; (4) a convolute surface, in which case successive positions of a straight edge will be tangent to a helix.

If a surface is *not ruled*, it will be a *double-curved surface*. If the double-curved surface is a *surface of revolution*, it may be identified by the fact that planes perpendicular to the axis of revolution will cut *circles* from the surface.

51. Surfaces are represented on a drawing and are located as to position in space by two views (at least) showing a *section* and the *limiting* or *contour* lines of the surface as orthographically seen from the top, front, and end. These sections may be *right* (perpendicular to the center line) or oblique (at an angle to the center line) and identify surfaces as right or oblique surfaces.

In the next few pages are illustrated a number of surfaces of each class, and the graphic method of describing these is shown on the drawings which accompany the description.

52. **Representation of Surfaces.** Surfaces are represented by two or more views of some *section* of the surface and *contour lines* of the surface.

A section is an area cut from a surface by any plane passing through the surface. This cutting plane may be (1) at right angles to the elements, or to the axis; or (2) oblique to these. In the first case the section cut by such a cutting plane is known as a *right section*; in the second case the oblique cutting plane cuts an *oblique section.*

Plane Surfaces. Top Drawing. Figure 37. A plane surface is made up, in drafting work, usually of a combination of three or more planes. In the object shown, four planes are combined to form a prism. The section shown is parallel to the top plane, and the edges of the prism are parallel to AB. It is especially to be noted in this and all representations of surfaces, that the limiting, or contour, elements of the *top* view are *not* projections of the limiting, or contour, elements of the front view.

Single-Curved Surfaces. Middle Drawing. Figure 37. Single-curved surfaces are ruled. Like plane surfaces, they are represented by two or more views of some section to indicate shape and some *line* (element) to indicate direction. In the object shown, an oblique view of a circular section with limiting or contour lines meeting in a point represents a cone. The limiting elements which represent the top view are *not* projections of the limiting elements of the front or the end view.

Warped Surfaces. Bottom Drawing. Figure 37. If a *ruled* surface is neither *plane*, nor *single curved*, it is, therefore, *warped.* There are a great variety of warped surfaces relatively few of which have practical applications. The one illustrated is a cylindroid and is represented by two circular sections connected by a series of elements which are all horizontal lines. In drafting, only the contour elements would be drawn. The position of these various elements suggests how a form could be built to form a vaulted ceiling of this shape.

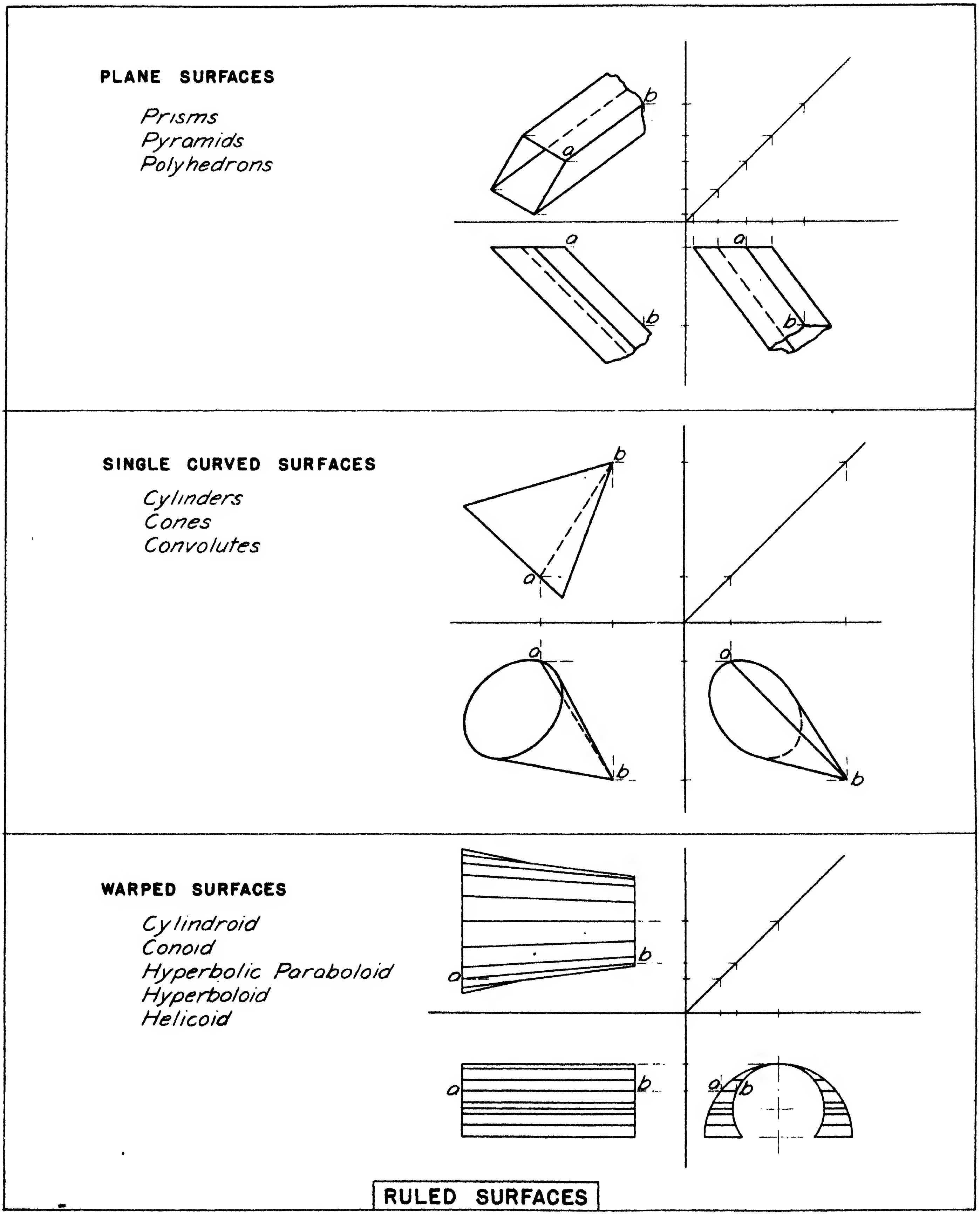


FIGURE 37
Representation of Surfaces.

52. **Represenation of Surfaces** (*cont.*). Surfaces of double curvature are commonly also surfaces of revolution. They may be generated either by the revolution of a curved line about an axis, or by the revolution of a straight line about an axis. Both classes of such surfaces are illustrated on the facing page.

Double-Curved Surfaces of Revolution. Top Drawing. Figure 38. If any *plane* curve be revolved on an axis, a surface of double curvature will be formed, and all sections of the surface perpendicular to the axis of revolution will be circles. In the figure shown, an ellipse has been rotated about its major axis, thus forming a prolate spheroid, or ellipsoid. The contour element in the top view is the largest *circle* on the surface, and the contour element in the front view is the largest ellipse on the surface.

Ruled Surfaces of Revolution. Bottom Drawing. Figure 38. If any straight line be revolved about an axis, there will be generated a surface of revolution. If the moving line remains parallel to the axis, a right cylinder will be formed; if the moving line intersects the axis, a right cone of two nappes will be formed; if the moving line neither remains parallel to nor intersects the axis, an hyperboloid will be formed.

This is the figure illustrated; the end view shows the characteristic contour, the front and top views show the successive positions of the moving line. These positions of the rulings indicate how teeth may be cut on such a surface to engage with teeth on a similar surface. A pair of such "rolling hyperboloids" make it possible to transmit motion between two shafts which are neither parallel nor intersecting.

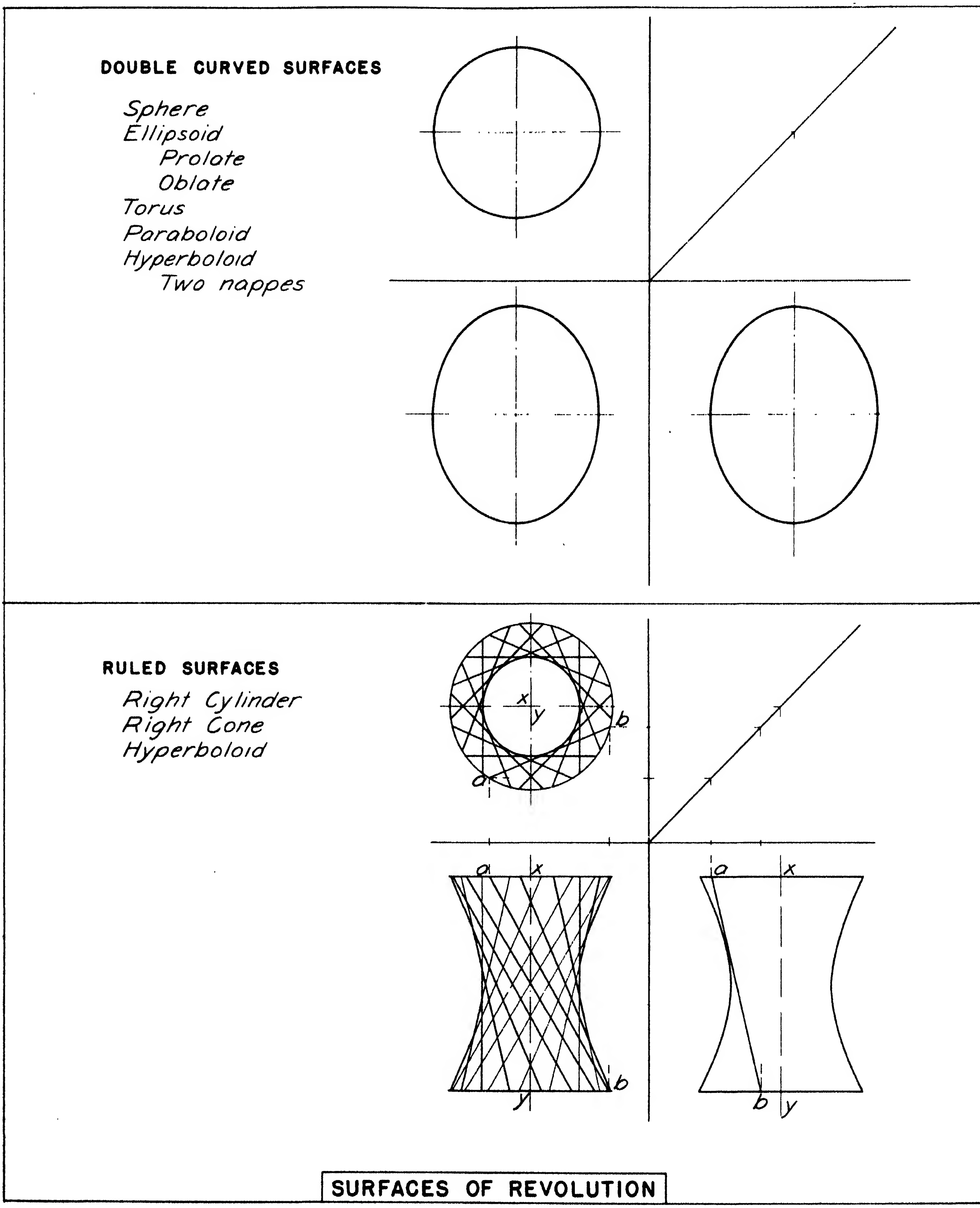


FIGURE 38
Representation of Surfaces.

53. EP13. To find the section cut from a surface by a plane.

General Method for All Ruled Surfaces. If the points where the rulings—or elements—of the given surface pierce the given plane are located (Article 40, Fig. 29), the shape and size of the required section will be shown.

For plane surfaces only the points where the edges of the surface pierce the given plane need be found; for single curved and warped surfaces a series of piercing points on elements closely spaced must be found in order to obtain an accurate and smooth curve of intersection.

Figure 39. An open oblique pyramid with an hexagonal base is cut by a plane *ABCD*. The true shape and size of the section cut from the pyramid by the plane are required. To find this section:

1. Locate RL_r perpendicular to direction line *CX* in plane *ABCD*, and find the auxiliary views of *ABCD* (an edge view) and of the pyramid.

2. This edge view of the plane *ABCD* shows where all elements of the pyramid enter the plane and, therefore, locates an edge view of the section.

3. Consider this plane of the section as an auxiliary plane and locate *RL*.

4. By projection and measurement from the front view of the section (the front view is obtained by projection from the edge view of the section) locate the fundamental view of the section on plane *RL*.

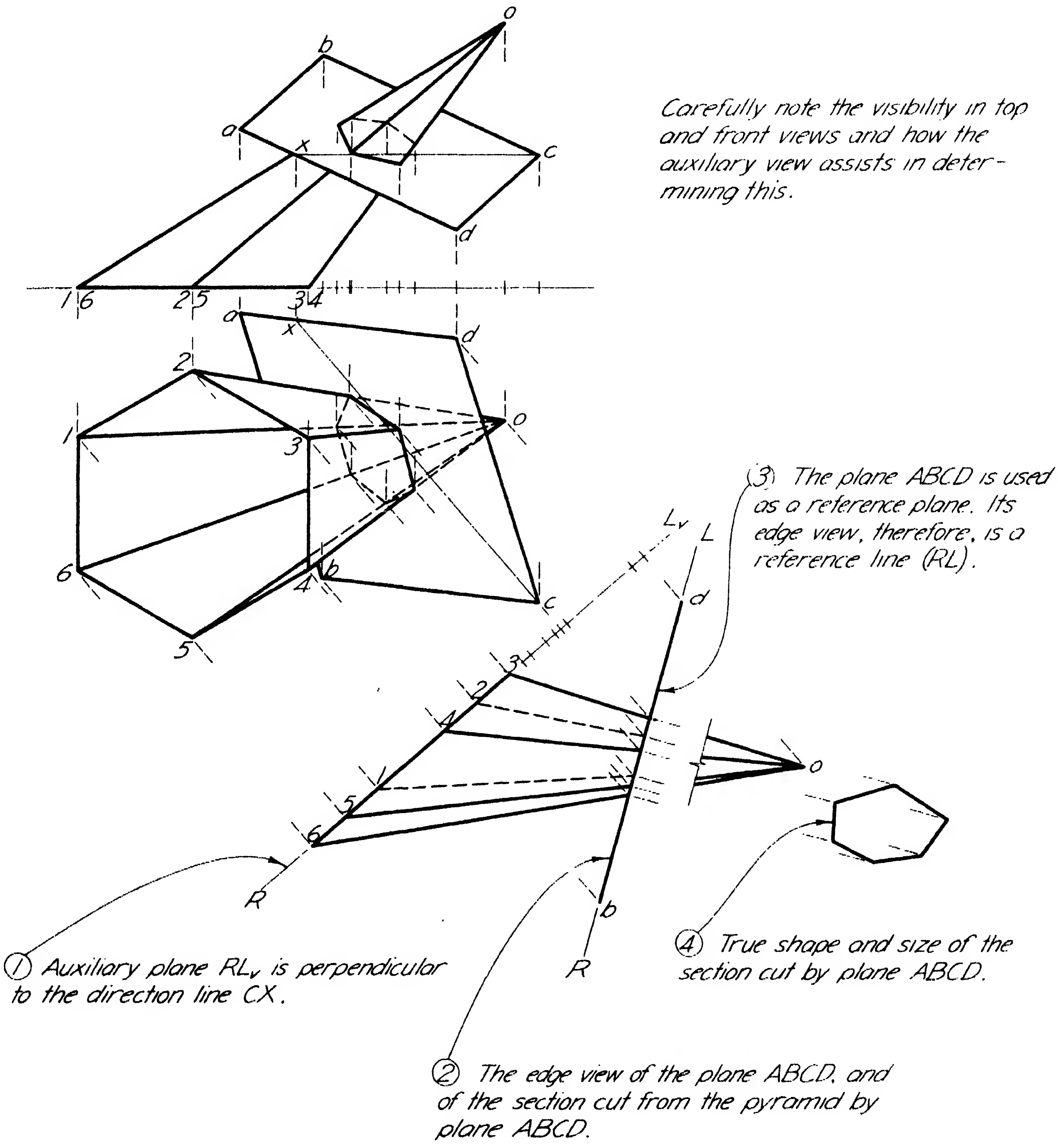
Visibility, and its correct representation by full and by dashed lines on a drawing, requires careful consideration. In this problem the pyramid is not a solid. Visibility, with respect both to the cutting plane and the given surface, can be determined by reference to the auxiliary view.

The exploration of which edges are visible, which edges are invisible, and which edges even though visible as elements of the object of which they are a part, are nevertheless covered and rendered invisible by other objects, can be accomplished by the study of *one view at a time.*

By concentrating on the top view, for example, and finding which are the nearest and farthest points from the *H* plane, the visibility of edges and surfaces in the top view becomes established. In like fashion, a study of the front view reveals that the elements of the object *nearest* the *V* plane control visibility in this view.

When the visibility of a single unit has become established, interferences with this established visibility by other units may then be explored.

The establishment of visibility is not only important in the correct representation of objects but its study contributes substantially to the development of that essential quality of the engineering mind known as "the power to visualize."



THE SECTION CUT BY A PLANE FROM A PLANE SURFACE

FIGURE 39
Essential Principle 13.

54. **EP13** (*cont.*). **To find the section cut from a surface by a plane.**

When the given surface is not ruled, the problem requires careful analysis and exceedingly precise drawing methods for an accurate solution. A series of auxiliary "slicing planes" should be chosen as to position so as to cut the simplest and easiest-to-draw lines from the curved surface. The method may be readily visualized by reference to the action of mechanical bread and bacon slicers.

Figure 40. The semiellipsoid is cut by a quadrilateral plane *ABCD*. The curve of intersection cut from the ellipsoid by the plane may be found by slicing the surface of revolution by a series of horizontal planes perpendicular to the axis. These slicing planes will cut circles from the ellipsoid and lines from plane *ABCD*. Since each circle and line are contained in a common slicing plane, they will intersect and locate points common to the plane *ABCD* and the ellipsoid.

To find the section:

1. Note the horizontal plane through X and Y. This slicing plane cuts a circle from the ellipsoid (see top view for the size and position of this circle) and a right line XY from plane *ABCD*. The line intersects the circle at points 1 and 2, which are points on the surface common to plane *ABCD* and the ellipsoid. They are, therefore, points on the section cut from the ellipsoid by plane *ABCD*. By passing a series of such slicing planes a sufficient number of points may be found to plot the curve of intersection.

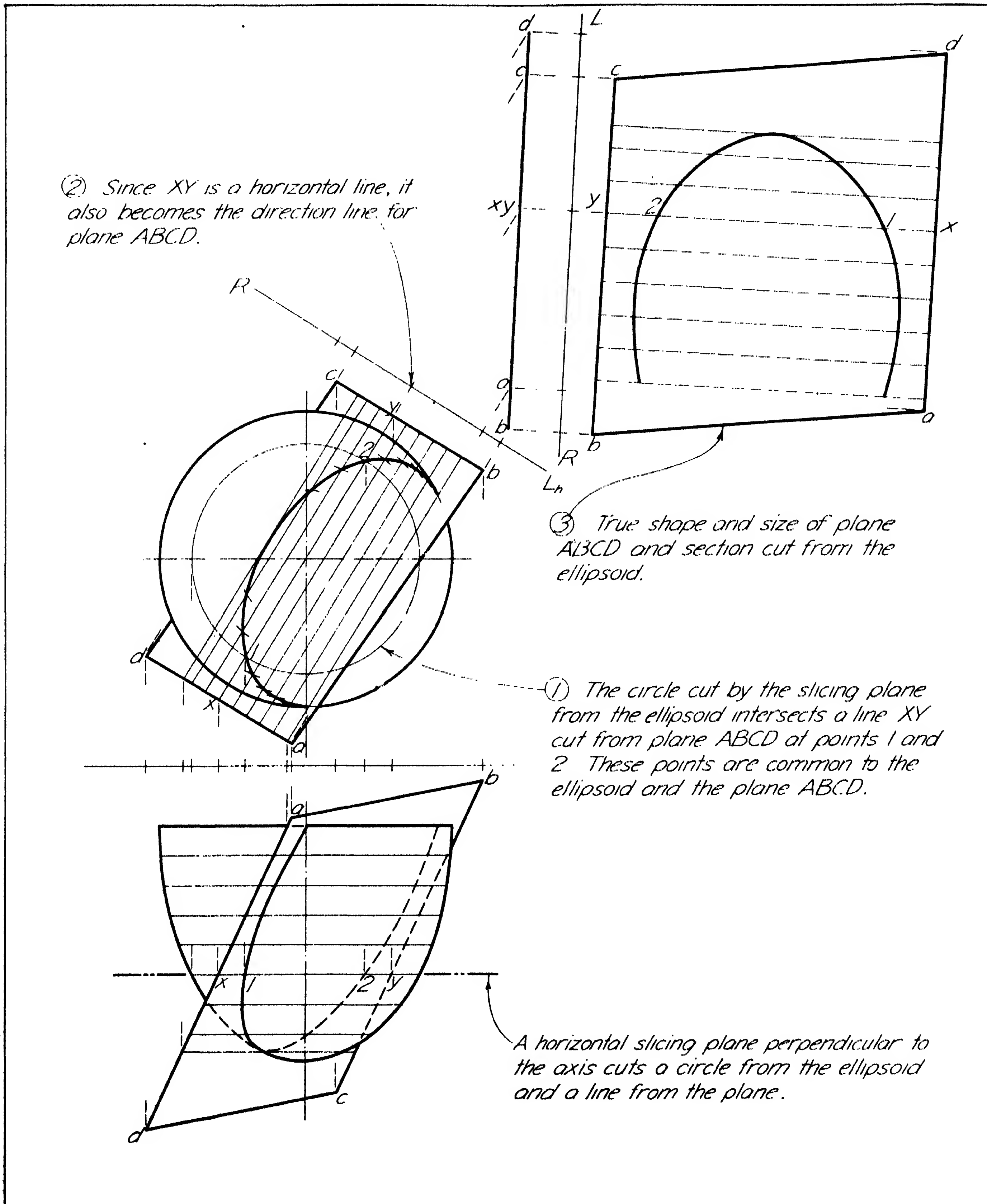
To find the true shape and size of this section:

2. An edge view of plane *ABCD* and the section is found on plane RL_h.
3. The true shape and size is located on plane *RL* which is parallel to this edge view.

The so-called *slicing-plane* method is useful in solving problems where lines of equal elevation, called *contours*, are used to represent levels on the surface of the ground, and the area is to be changed by cuts or fills.

This application may be comprehended by imagining a hill in the general shape of a cone. If this hill were intersected by a series of level planes all equidistant apart vertically, the top view of the hill could be represented by a series of more or less concentric curves each of which would represent a line of known elevation on the surface and each would be an equal elevation above or below its adjacent neighbor. Thus in a *one-view drawing*—known as a contour map—when elevations are given for each contour, it becomes possible to represent three dimensions.

When such a hill is intersected by a plane, or a prism, or by any other known surface, it becomes possible to show on this one-view drawing the intersection, and the modifications in shape of the hill, by applying the method of the slicing plane as shown in Figure 40.



THE SECTION CUT BY A PLANE FROM A SURFACE OF REVOLUTION

FIGURE 40

Essential Principle 13 (*cont.*).

55. **EP13** (*cont.*). **To find the right section of a surface.** A right section of a surface is a section in a plane *perpendicular to the axis or the center line of the surface.*

Top Drawing. Figure 41. A prism oblique to H and V is given; to find the true shape and size of its right section. Since a right section will be perpendicular to the axis of the prism:

1. Locate a fundamental view of the prism. In this view of the prism, the plane cutting a right section will be perpendicular to the axis and will appear as an edge view.

2. On an auxiliary plane parallel to the edge view, derive the fundamental view of the section. This view will be the true shape and size of a right section.

Bottom Drawing. Figure 41. An oblique cone is shown with a circular base in V. To find its right section *at any chosen point*:

1. Draw the auxiliary view of the cone on a reference plane parallel to its center line. Since a right section at any given—or chosen—point will be perpendicular to the axis of the cone, bisect the angle at the apex in the auxiliary view. The right section will be perpendicular to this axis. This auxiliary view of the axis will coincide with an auxiliary view of the two cone elements which pass through the ends of one diameter of the right section.

2. Draw the edge view of the plane of the section perpendicular to this axis at the point where the section is to be taken.

3. Since one diameter of the right section will be the length of the edge view included between the limiting elements of the cone and the second diameter perpendicular to the first will appear as a point in this edge view, draw cone elements OC and OD. The front view will show how long CD—the second diameter—is.

By revolving the section about one axis, its true shape and size may be drawn. This "revolved view" is a common practice in showing, in a limited space, the true shape of sections. The right section of a pulley arm, or of a rib, is usually shown by this method.

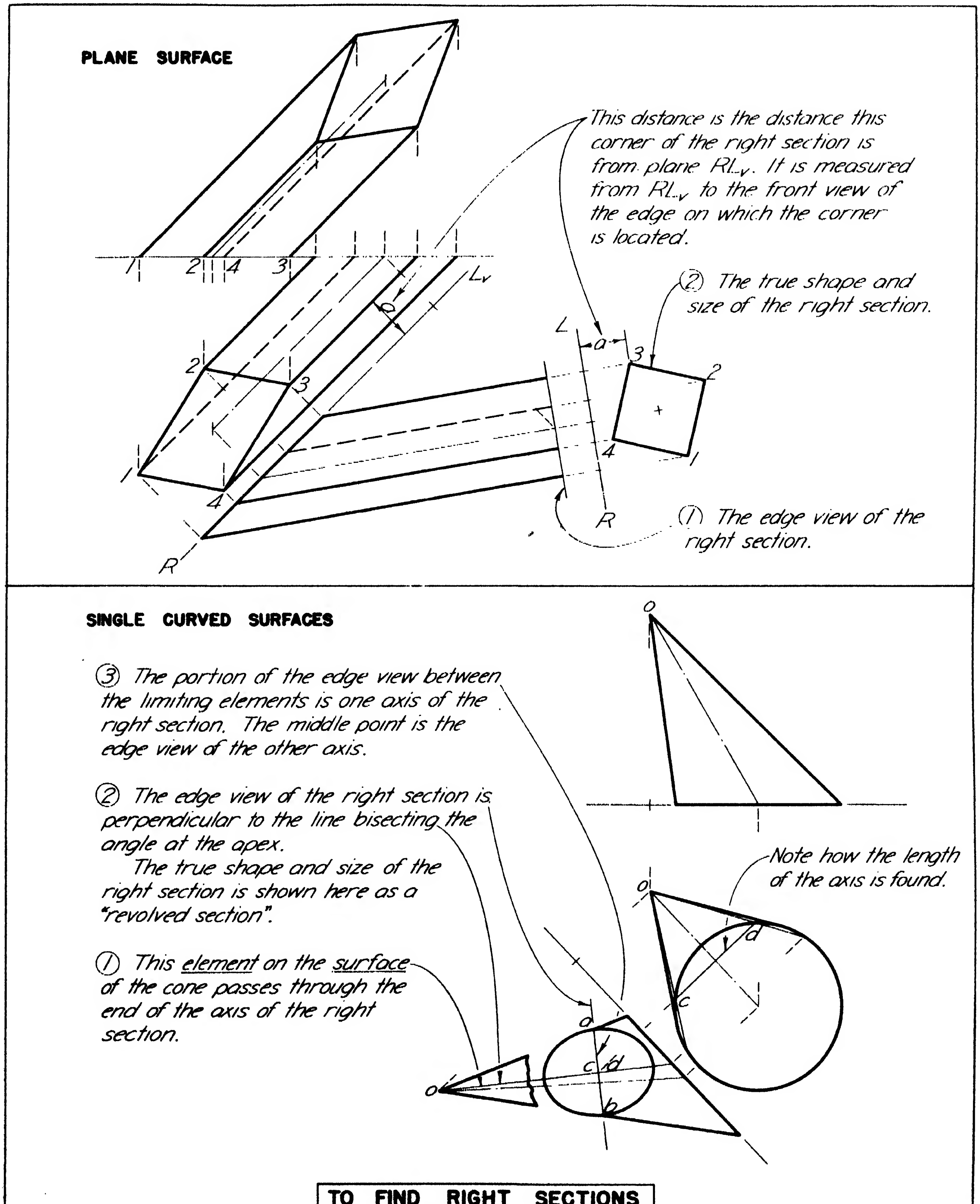


FIGURE 41
Essential Principle 13 (*cont.*).

56. EP14. To find the points where a line pierces a surface.

Two general methods are available which will apply to any problem. Both these methods are illustrated on the facing page. In the first method, a "slicing" plane is located perpendicular to V and containing the line. The shape the slicing plane cuts from the given surface is found. Where the given line cuts this section of the surface are the piercing points required.

In the second method, an auxiliary view of the surface on a plane parallel to the line is located. Since this view of the line is a fundamental view, the points where the line cuts the *auxiliary* contour outline of the surface will be the piercing points.

Top Drawing. Figure 42. A solid pyramid is pierced by the line *MN*. To find the piercing points:

1. Locate auxiliary plane RL_v perpendicular to V and containing line MN. The points 1234, where the edges of the pyramid cut this plane RL_v, locate the edge view of the section cut from the pyramid by this plane. See Articles 40 and 53.

2. By projection, locate the top view of this section and find points a and b in this view. These two points A and B, being common to the line MN and to the surface of the pyramid, show where MN enters and leaves the surface of the pyramid.

Bottom Drawing. Figure 42. A solid sphere is cut by a line MN. To find where the line pierces the sphere:

Locate an auxiliary plane RL_h parallel to line MN. Consider that a slicing plane containing MN and perpendicular to H cuts the sphere. Draw, on reference plane RL_h, the auxiliary view of MN and of the circle cut by the slicing plane from the sphere. Locate C and D—in the auxiliary view, the top view, and the front view—the points where MN cuts the sphere.

Note that the center of the circle cut by the slicing plane is point X, and the diameter of the circle is that portion of line mn in the top view which lies *inside* the top view of the sphere.

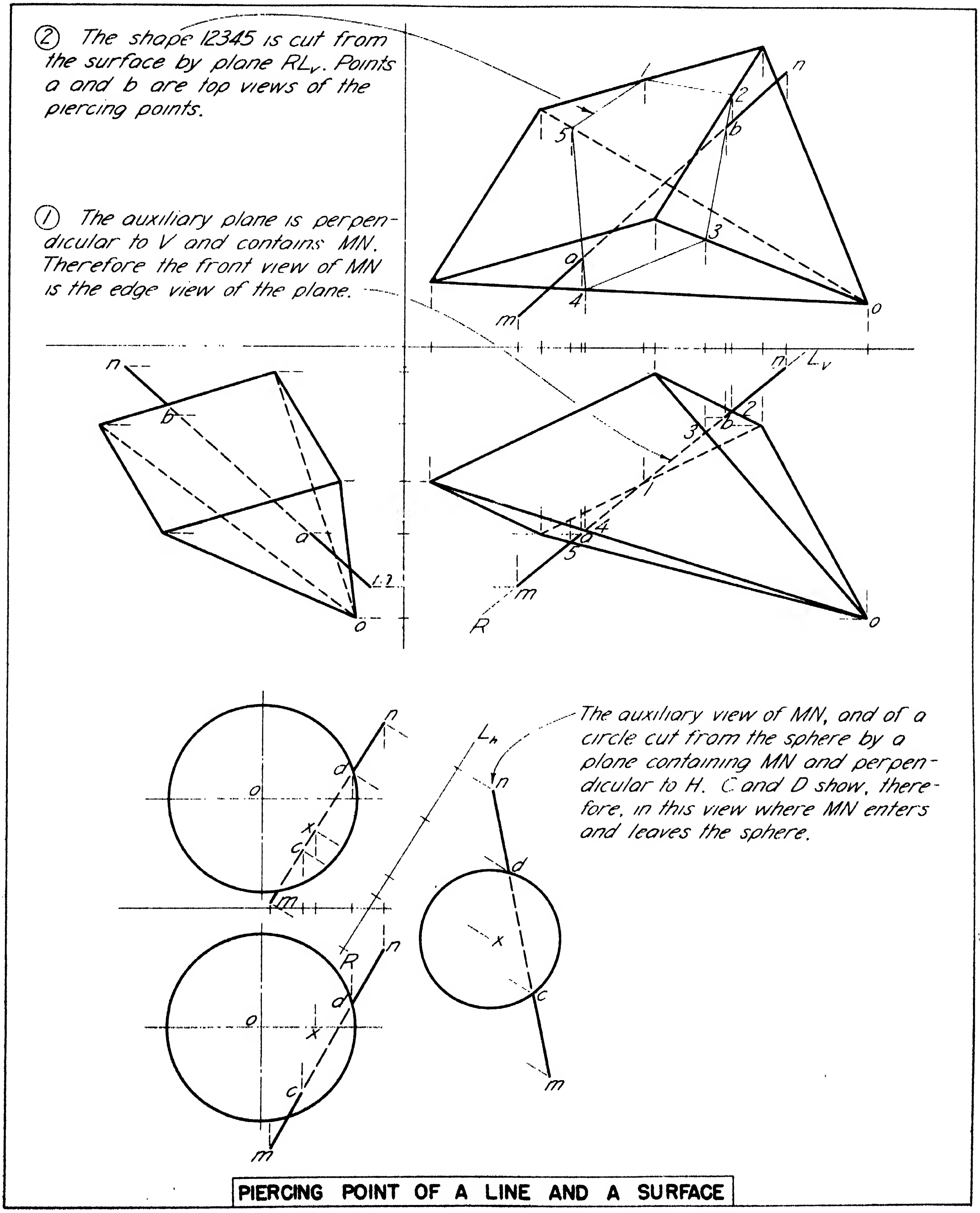


FIGURE 42

Essential Principle 14.

57. EP15. Given the size and position of a right section and the location of a center line; to draw any surface.

Figure 43. The location of OP, the center line, and the *size* of a right section of a timber are given. It is also specified that the *narrow edge* of this right section is to be *horizontal*. The problem requires a drawing of the timber and the shape and size of the faces of the timber in the top and front planes.

To draw the timber:

1. Locate RL_v parallel to the center line and find a fundamental view of the center line. Locate RL perpendicular to this fundamental view of OP and find op, the view of this center line, as a point on RL. See Article 55.

2. Locate a horizontal line AX of any length through any assumed point on OP. Make the fundamental top view of AX perpendicular to the top view of OP. AX, then, becomes a horizontal direction line in the plane of the right section. (The plane of a right section must be perpendicular to line OP.)

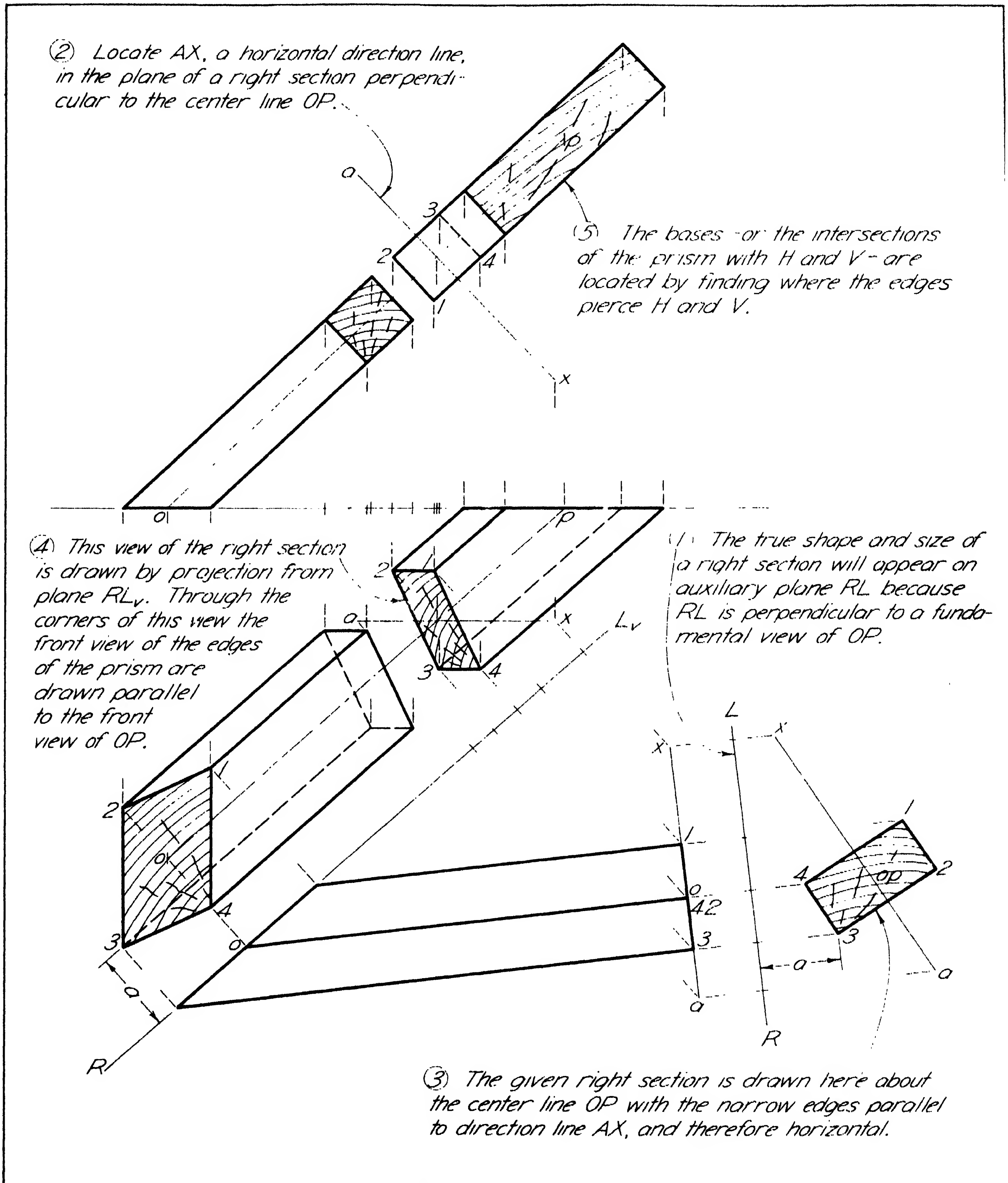
3. About op, on plane RL, as a center point, and making 12 and 34, the narrow edges, parallel to the horizontal line AX, construct the true shape and size of the right section according to given size and shape specifications. See Articles 38 and 39.

4. 1234, a view on plane RL_v of this right section, may now be drawn, and from this view on plane RL_v the front and top views of this section may be drawn by projection and measurement. Through the front and top views of 1234, the specified right section, the edges of the prism may be drawn parallel to the given center line OP.

5. Where these edges enter the top and front planes may now be located and the faces of the prism in H and V drawn.

Observe how visibility in the front and top views is established by reference to the view on RL_v.

It is to be noted that the front and top views of the right section are not required in order to draw the front and top views of the timber. Since points 1234, the corners of the right section, lie on the edges of the prism, and since the direction of these edges is parallel to the given center line, the base in V, as indicated by points 1234 on this base in the drawing, may be located by projection from RL_v and by measurement from RL. Note the dimension a in the drawing, and also refer to Article 55.



TO LOCATE A SURFACE FROM GIVEN RIGHT SECTION AND CENTER LINE

FIGURE 43
Essential Principle 15.

58. **EP15** (*cont.*). **Given the size and position of a right section and the location of a center line; to draw any surface.**

When the given section is a curve, consideration must be given to methods which will make the construction of the curve as simple and accurate as possible. In drawing constructions in general, the choice of method should always be made with the element of precision as an essential factor.

Figure 44. The axis OP of a cone is given. The right section of this cone at point P is a circle of given diameter. To locate and draw the top and front views of the cone:

1. On plane RL_v parallel to OP draw a fundamental view of the axis.
2. On plane RL, perpendicular to this fundamental view of OP, draw op, the point view of the axis; construct on plane RL the given circular right section with center at OP. Since a right section is perpendicular to the axis, the view of this circular right section on plane RL_v will be an edge view at p. The length of this edge view is equal to the diameter of the given right section. Since there are now two views of the right section, the third view on the front plane may be located by projection and measurement from these two views on RL_v and RL.
3. From the front view and the view on plane RL_v, a top view may now be constructed, and the limiting elements of the cone drawn to complete the top and front views.

These projection and measurement methods are laborious and not easy to make precise. A simpler and more direct method of finding the elliptical bases in front and top views is shown in the drawing. By projection methods the *axes* of the elliptical bases are located, and on these axes the curve of right section is constructed by the trammel method.

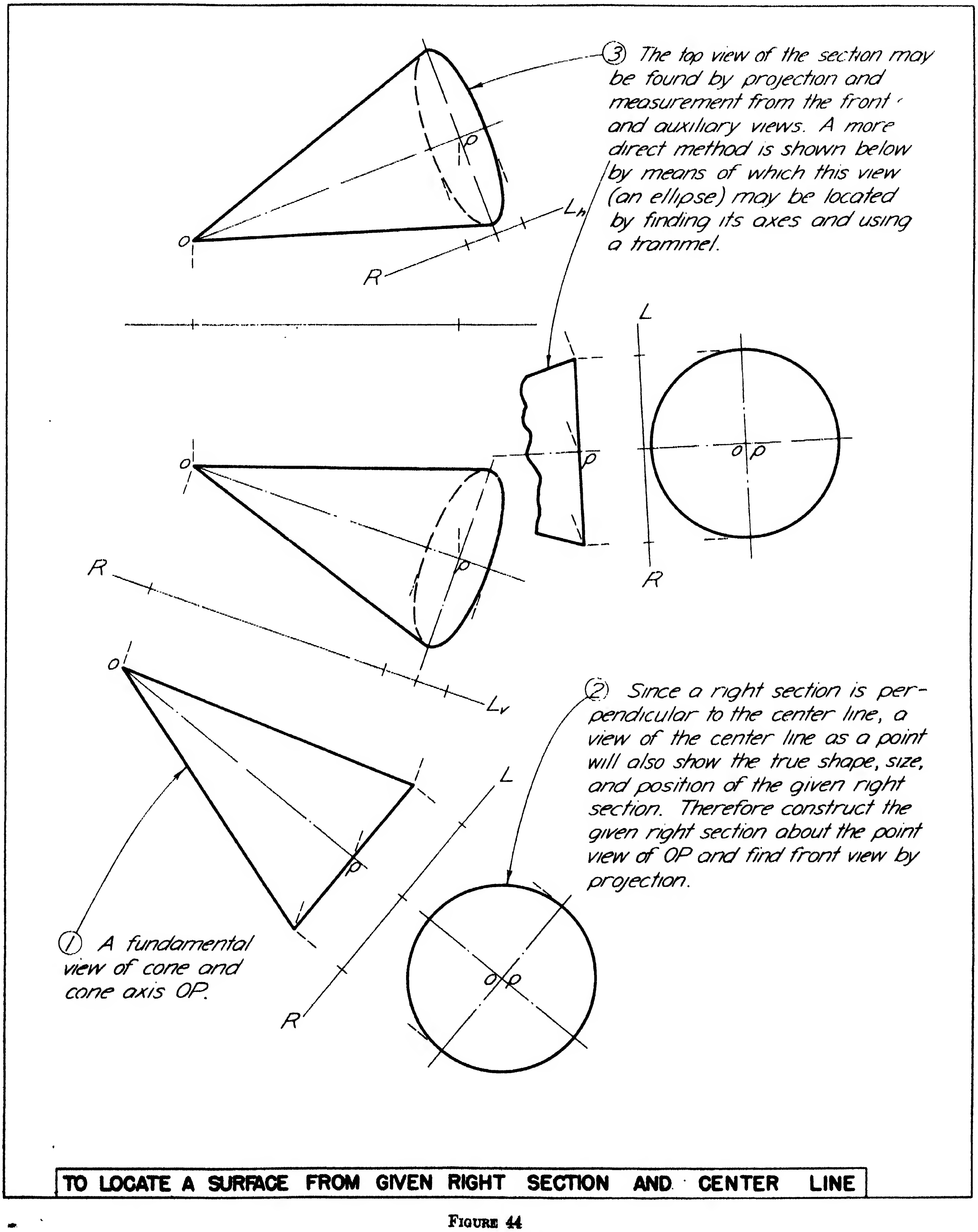


FIGURE 44

Essential Principle 15 (*cont.*).

59. EP16. To find the intersection of two surfaces.

The methods of solving problems in the intersection—sometimes called the interpenetration—of surfaces are found in these essential principles: (1) finding where lines of one surface enter the second surface (Article 56, Fig. 42); (2) finding where planes of one surface cut the second surface (Article 46, Fig. 35); (3) a combination of the first two methods. The choice of method depends on the nature of the surfaces and especially upon the simplicity and precision of the drafting operations. In the problems which follow, a variety of surfaces are illustrated with intersections located by a number of methods.

Two Plane Surfaces. Figure 45. An open square prism and an open hexagonal pyramid are shown by top and front views. To find the intersection of the two surfaces:

1. Locate on reference plane RL_v (drawn perpendicular to the front fundamental view of the prism) an auxiliary view of the pyramid and an "edge" view of the prism. These auxiliary views show where all edges of the pyramid enter the prism. The front and top views of these piercing points may be located in the front and top views of the surfaces by projection. When the points are joined in proper order the intersection is thus described.

2. Observe that the top edge of the prism enters the horizontal hexagonal base of the pyramid. The top view shows this situation. Therefore two edges of the horizontal base enter the top slanting faces of the prism. The top view shows this situation as well as the front view. The location of the piercing points on the slanting faces may be located by means of auxiliary elements. The location of these elements is fixed by use of the auxiliary "edge" view of the prism. From this view, these elements may be projected to the front and top views and the piercing points located.

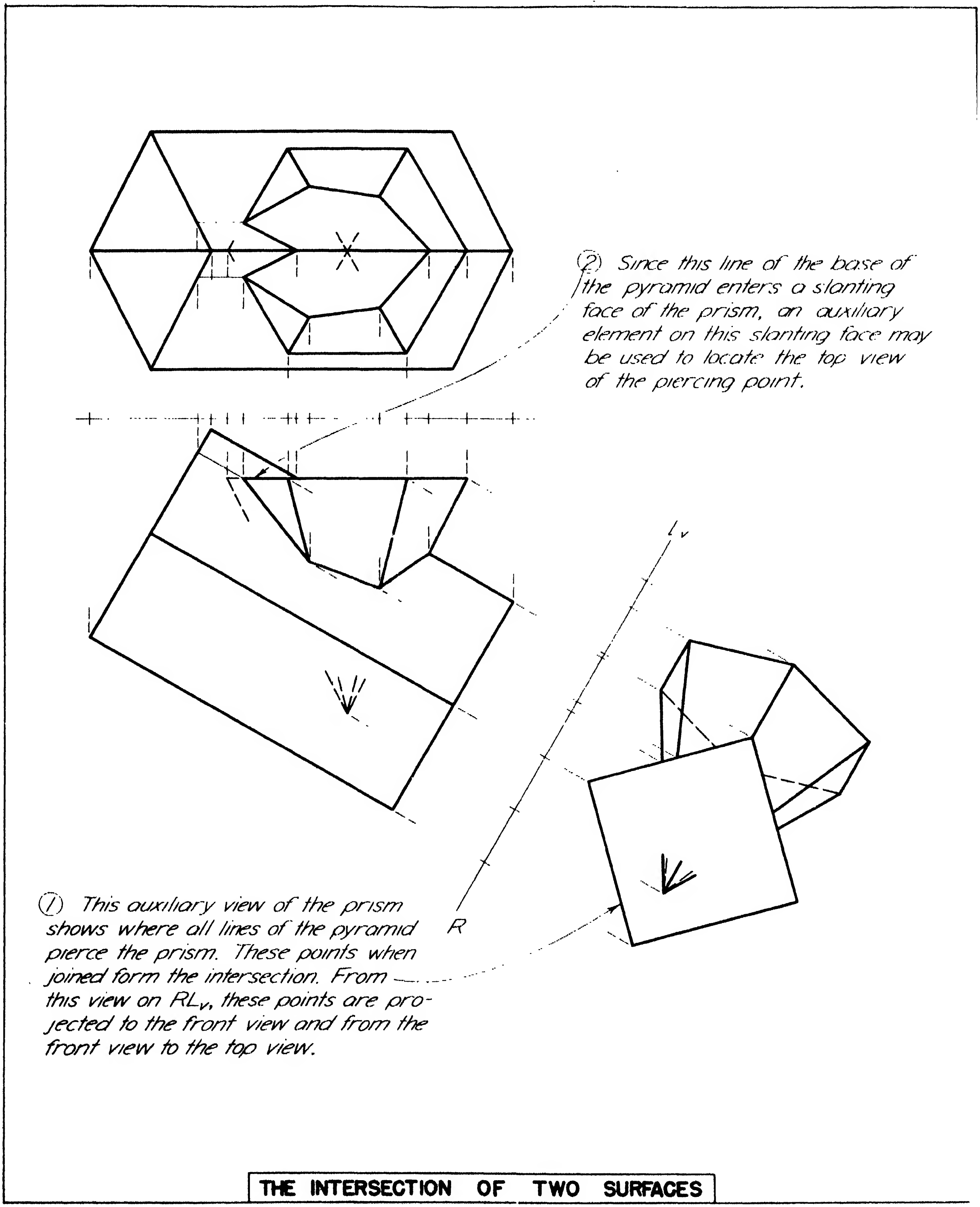


FIGURE 45
Essential Principle 16.

60. EP16 (*cont.*). **To find the intersection of two surfaces.**

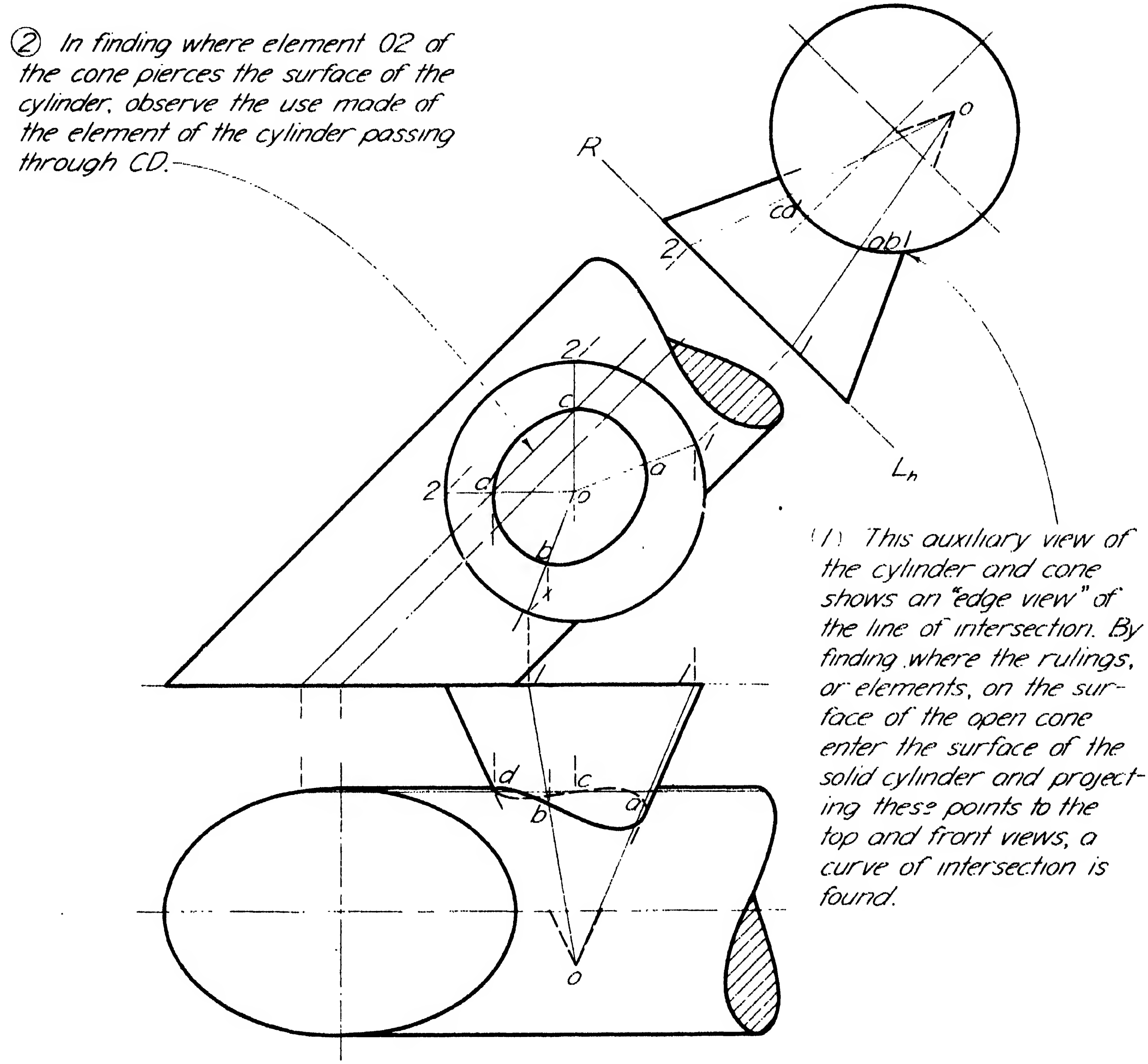
In drawings requiring many points to be located in order to secure a curve of intersection, systematic construction methods are essential; otherwise the many necessary construction lines will cause confusion and thus error. In most problems of this sort it is best to follow through each point at a time and to secure *all its views before additional construction lines are drawn*. Also it is desirable to begin at one side of the object and work in one direction systematically.

In some of the problems which follow, only enough construction is drawn to illustrate method and principle; complete construction would require so many lines that the figure would become unnecessarily involved.

Single-Curved Surfaces. Figure 46. The top and front views of an open cone and a solid cylinder are given. To find their curve of intersection:

1. Locate plane RL_h perpendicular to the top (fundamental) view of the cylinder. On this reference plane locate an auxiliary view of the cone and an "edge" view of the cylinder. All points common to the two surfaces will appear on the "edge" view of the cylinder between the limiting elements of the cone in this view. This auxiliary view of the intersection of the two surfaces may be projected to the top and front views of the surfaces by the use of elements.

2. For example: Draw cone element *o*2 in the auxiliary view. Point 2 of the base will have two possible positions, and will therefore locate points *C* and *D* on the element *CD* of the cylinder. The plan view of these points is located as shown on the drawing. The elevation view may be found either: (*a*) by locating the elevation view of cone elements *o*2 and finding front views of *C* and *D* by projection, or (*b*) by locating the elevation view of cylinder element *CD* and finding by projection where *c* and *d* are located on this line. The factor of graphical precision should influence the choice of method.



THE INTERSECTION OF TWO SURFACES

FIGURE 46
Essential Principle 16 (*cont.*).

61. **EP16** (*cont.*). **To find the intersection of two surfaces.**

In nearly all problems involving ruled surfaces, the method of finding where rulings, or elements, on one surface enter the other suggests itself. The elements are usually drawn on that surface which does not appear as a fundamental view, and finding where these elements enter the surface which does appear as a fundamental view.

Single-Curved Surface and Plane Surface. Figure 47. The open square prism and an open cone are shown by top and front views. To find their curve of intersection:

1. On plane RL_v perpendicular to the front (fundamental) view, locate an "edge" view of the open prism and an auxiliary view of the cone. Observe that the base of the cone in the auxiliary view may be drawn by the trammel method. In this view appear, as at x and y, all points where the elements of the cone, as $o2$ and $o1$, enter the surface of the prism. Having located x and y in the auxiliary view, locate the elements $o2$ and $o1$ on the cone by projection on the front view. By projection from the auxiliary view locate the front views x and y on the corresponding front view of the appropriate element $o2$ or $o1$.

2. To locate x and y on the top view, the element AB on the surface of the prism should be used. The location of AB is more readily found than the location of the top view of $O1$ and $O2$ of the cone because of the position of the base of the cone.

A sufficient number of points closely enough spaced to locate a smooth curve may be found by this method to determine the curve of intersection.

The visibility of the curve of intersection when the open cone is truncated by the planes of the open prism, of the limiting elements of both cone and prism, and of the sections of the cone and prism may be determined by referring to the auxiliary view and by the suggestions concerning the determination of visibility on page 84.

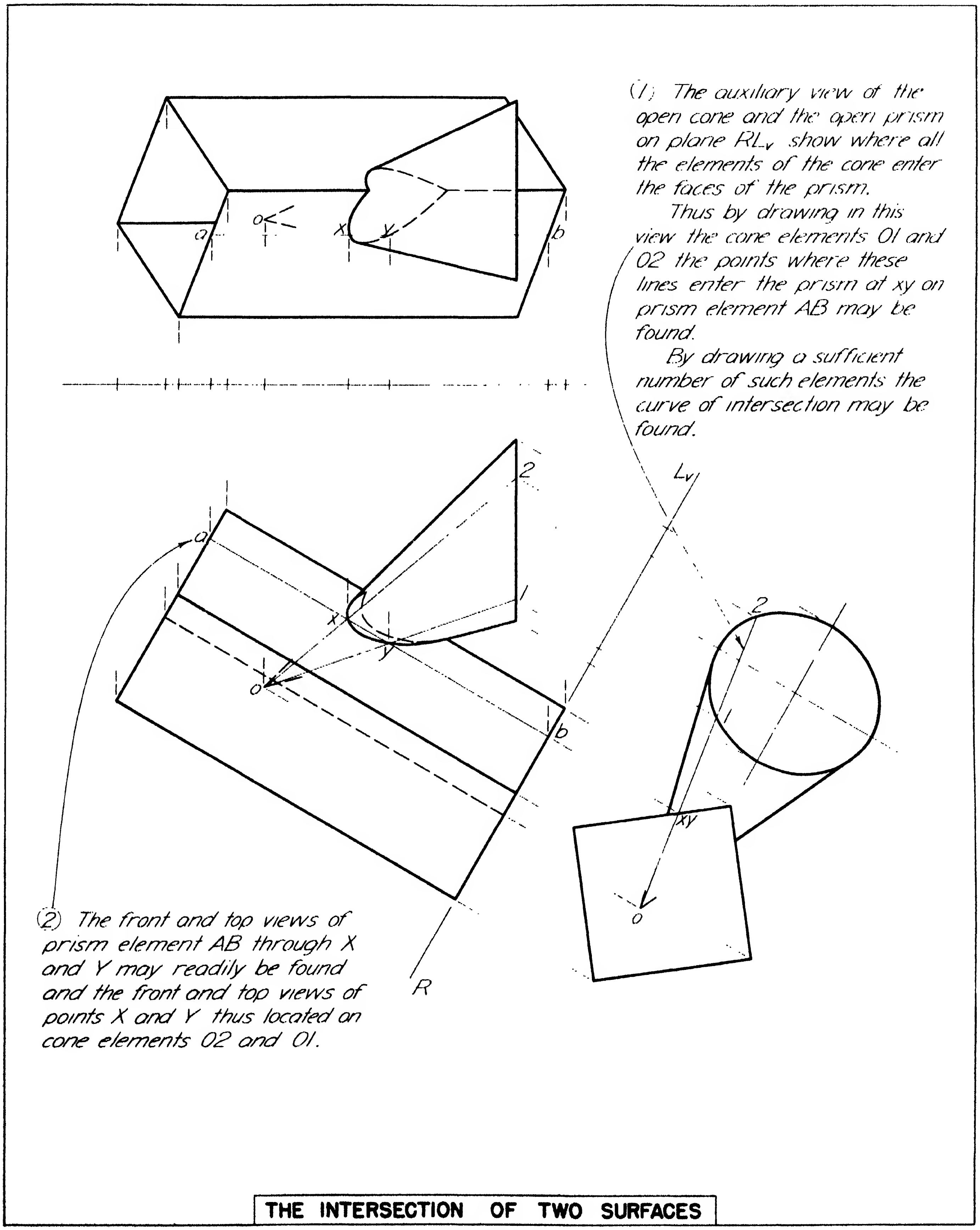


FIGURE 47
Essential Principle 16 (*cont.*).

62. **EP16** (*cont.*). **To find the intersection of two surfaces.**

In problems where one or both surfaces are *not* ruled, consideration must be given to methods making the problem as simple to draw and as precise as is necessary. No general method can be given which will apply to all problems. The nature of the surfaces, and their positions with respect to each other, must be studied, and the most satisfactory method devised for each case.

Single-Curved Surface and Surface of Revolution. Figure 48. A study of the oblate ellipsoid and right cylinder which are shown discloses that no previously illustrated method is easy to apply. The simplest element to be cut from the ellipsoid is a circle. Planes cutting such circles will have to be perpendicular to the axis of revolution. Such perpendicular slicing planes will cut ellipses from the cylinder. These ellipses from the cylinder and circles from the ellipsoid will intersect each other in points on the curve of intersection. Analysis reveals the fact that, though this method seems complicated, no other method seems any simpler.

To find the curve of intersection:

1. Horizontal slicing plane 1 cuts a circle from the ellipsoid.

2. This same slicing plane also cuts from the cylinder an ellipse whose axes will be projected on the top view in their true length and relation.

3. In place of drawing the many ellipses required for a complete curve of intersection, a trammel may be used (all the ellipses will be the same size) to plot the points where these ellipses intersect the circles of the ellipsoid cut by a series of slicing planes parallel to slicing plane 1.

4. By using a sufficient number of slicing planes parallel to slicing plane 1, a series of circles and ellipses will be cut from the two surfaces, thus locating enough points on the curve of intersection to draw a smooth and accurate curve.

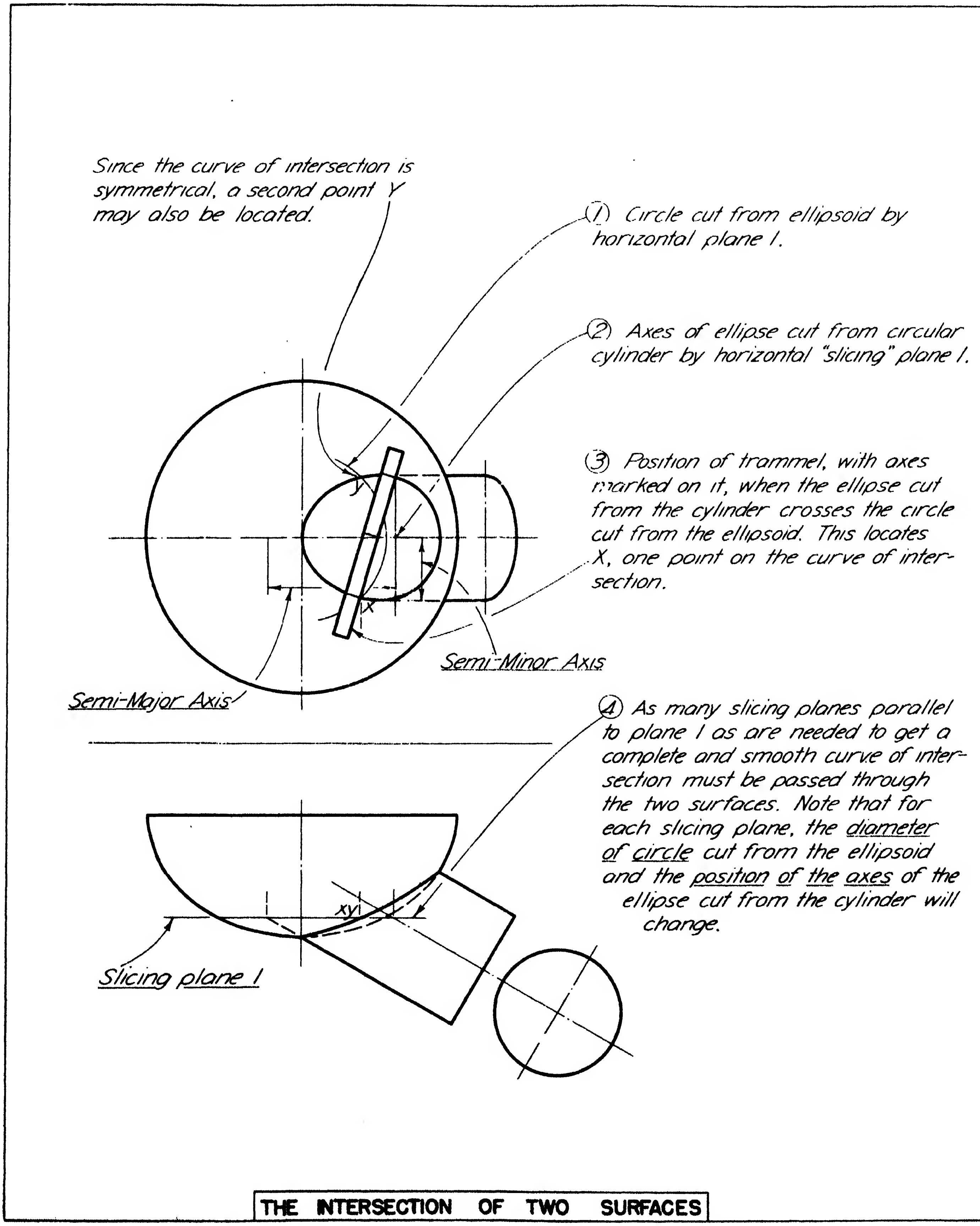


FIGURE 48

Essential Principle 16 (*cont.*).

63. EP16 (*cont.*). **To find the intersection of two surfaces.**

In all cases of EP16 previously illustrated at least one surface was in a fundamental position parallel to one of the principal planes of projection. When such is not the case, and both surfaces are oblique to the principal planes, double auxiliary projection must be used in order to place one of the surfaces in a fundamental position. Such a case is illustrated in Fig. 49.

Two Oblique Surfaces. Figure 49. Both the given pyramid and cylinder are solids and are shown by top and front views. To find the curve of intersection:

1. Locate by double auxiliary projection an "edge" view of the elliptical cylinder and an auxiliary view of the pyramid on auxiliary plane RL. (Auxiliary plane RL is perpendicular to the fundamental view of the cylinder on plane RL_h which is parallel to the cylinder.) Find where elements on the faces of the pyramid enter the cylinder as at x on plane RL.
2. By projection locate x on plane RL_h.
3. From the view of x on plane RL_h, and by projection of element AP onto the top and front views, the position of X in the top and front view may be located. By using a sufficient number of elements similar to AP, a complete intersection may be found.

The visibility of lines must be carefully studied. Even though a point may be visible so far as one surface is concerned it may be hidden by the other surface. Furthermore, points visible in one view may not necessarily be visible in other views. The limiting, or contour, elements of the surfaces are useful in determining visibility. For example, in Figure 49, a study of the front view of the cylinder shows that *no element* of this cylinder on the *far side* of the two contour, or limiting, elements can be visible in the *front* view. In like fashion, a study of the top view shows that *all elements* on the *left* side of the transverse diameter of the base in H are visible unless they are covered by the pyramid. Thus, the visibility of the cylinder is controlled in part by the visibility of the pyramid. The method of discovering the visibility of the pyramid is discussed on page 84, and may be accomplished by the study of one view at a time and the determination of the near and far elements of the object as seen in that view.

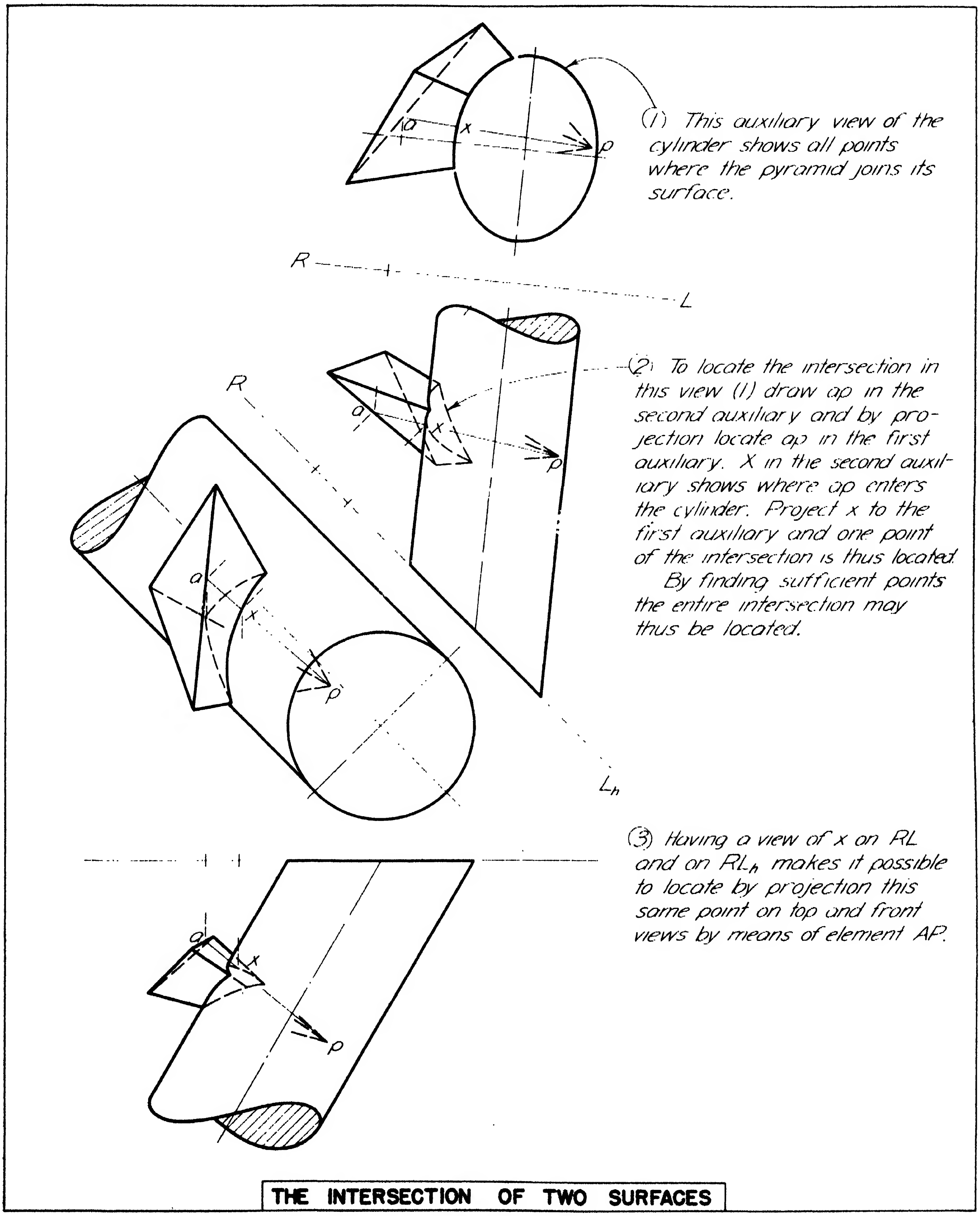


FIGURE 49

Essential Principle 16 (*cont.*).

64. **EP16** (*cont.*). **To find the intersection of two surfaces.**

A common problem in surfaces is the design of elbows and breechings so that they are made up of surfaces which have circular sections and whose joints are plane curves. Since a cone or a cylinder tangent to a sphere, whose center is on the axis, will have a circular section in the plane of tangency, this principle is used in such design.

Cylindrical Elbow. Top Drawing. Figure 50. The diameter of the circular openings is given, and the elbow is to turn a right angle. An elbow may have more than two or three sections, but the principle of locating each section by making it tangent to a sphere of the diameter of the openings is the same. The center of the sphere must be on the axis of the tangent cylinder and, therefore, at the intersection of axes. The curve of intersection will appear as an edge view joining the limiting, and tangent, elements.

Conical (Reducing) Elbow. Middle Drawing. Figure 50. The position of the axes of the end sections and the diameter of their openings are given. The position of the axes of the intervening sections is adjusted according to the number of sections and the proper rate of turning the given angle between end sections. Spheres, of a diameter selected to secure uniform and appropriate reduction in diameter, are drawn at the intersection of the axes, and the limiting elements are made tangent to these spheres.

Breeching. Bottom Drawing. Figure 50. This same principle may be used in designing a breeching to connect two smaller cylinders with a larger cylinder by right cones. The surfaces are made tangent to spheres drawn at the intersection of the axes.

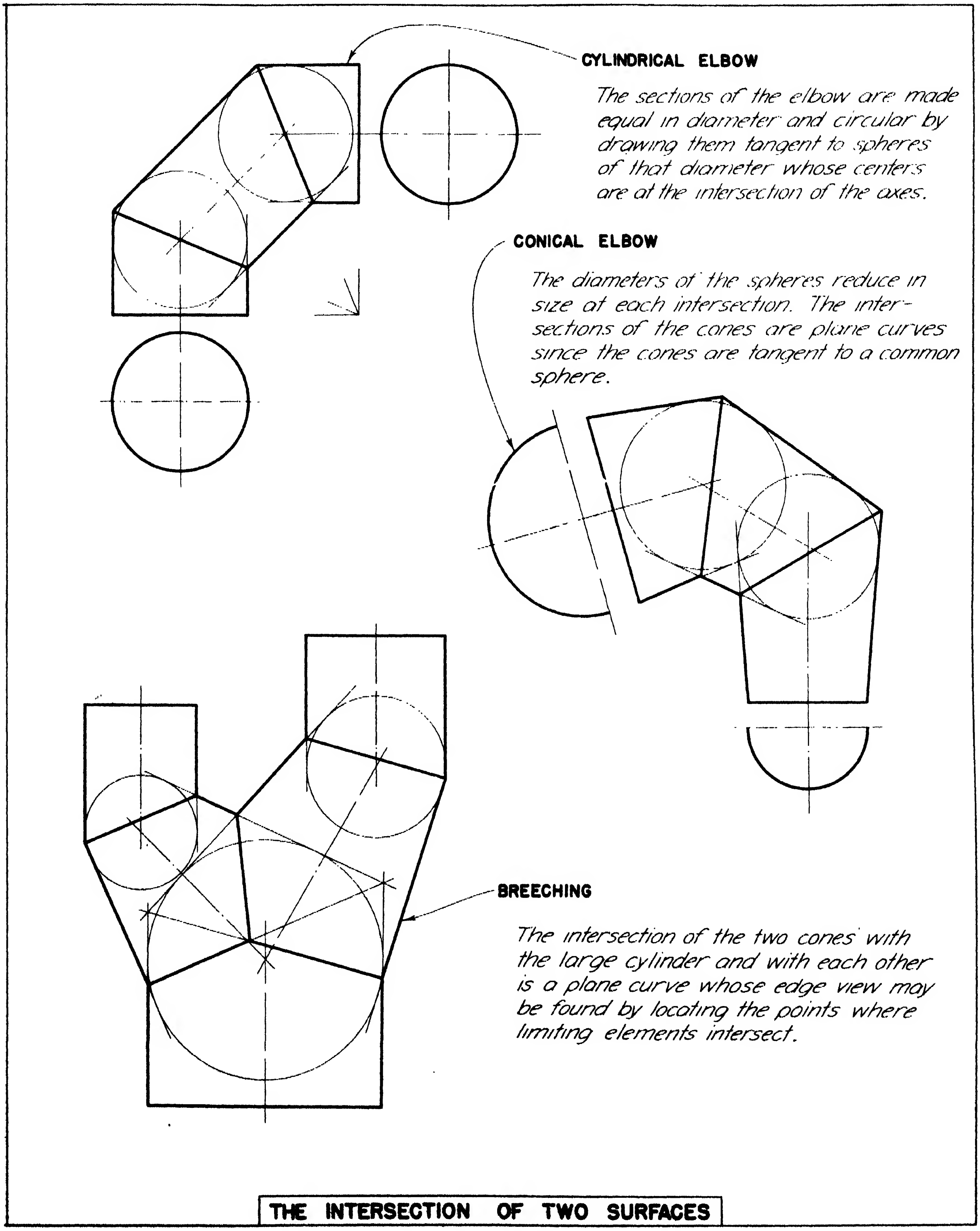


FIGURE 50
Essential Principle 16 (*cont.*).

65. EP17. To lay out a pattern for a surface.

Patterns, or templates, are used in laying out shapes for sheet-metal work such as tanks, elbows, bins, and chutes. These forms are constructed by draftsmen to enable workmen to mark out on unformed material the shape in which it is to be cut so as to form the several parts. When these parts are fastened together the surface will take the shape designed and will fit into its appointed place.

Plane Surface. Figure 51. A chute running between an opening in a floor and an opening in a wall is illustrated. To lay out a pattern:

1. Find a fundamental view on plane RL_v and a right section on plane RL.

2. Since the right section will develop into a straight line perpendicular to the edges of the chute, draw the line 23412 perpendicular to the fundamental views of the edges as shown on RL_v. Mark off on this line—always split the pattern on a short edge—the distances 23, 34, 41, and 12 as obtained from the true shape and size of the right section. Through each of these points draw a line parallel to the fundamental view of the edges on plane RL_v (these lines will also be perpendicular to the developed right section line 22). By projection from the fundamental view lay off the true lengths of these edges and connect the terminal points in order. When folded and bent so as to have a section of the shape of the right section, this pattern will form the shape of the chute.

It is to be noted that neither the auxiliary view on plane RL_v nor the elevation view of the right section is needed for this development. These were drawn on the problem for description and explanatory purposes.

When cutting and folding patterns to form models—and students are advised [illegible] do this—allowance should be made for lap on the split edge. For practical reasons, this joint sh[illegible]ld be along the shortest edge. Tabs and slits as illustrated by folding models Figs. 13*a*, 14*a*, and [illegible] also may be used. As a means of securing a true edge when folding, the pattern may be scored on the outside of the bend.

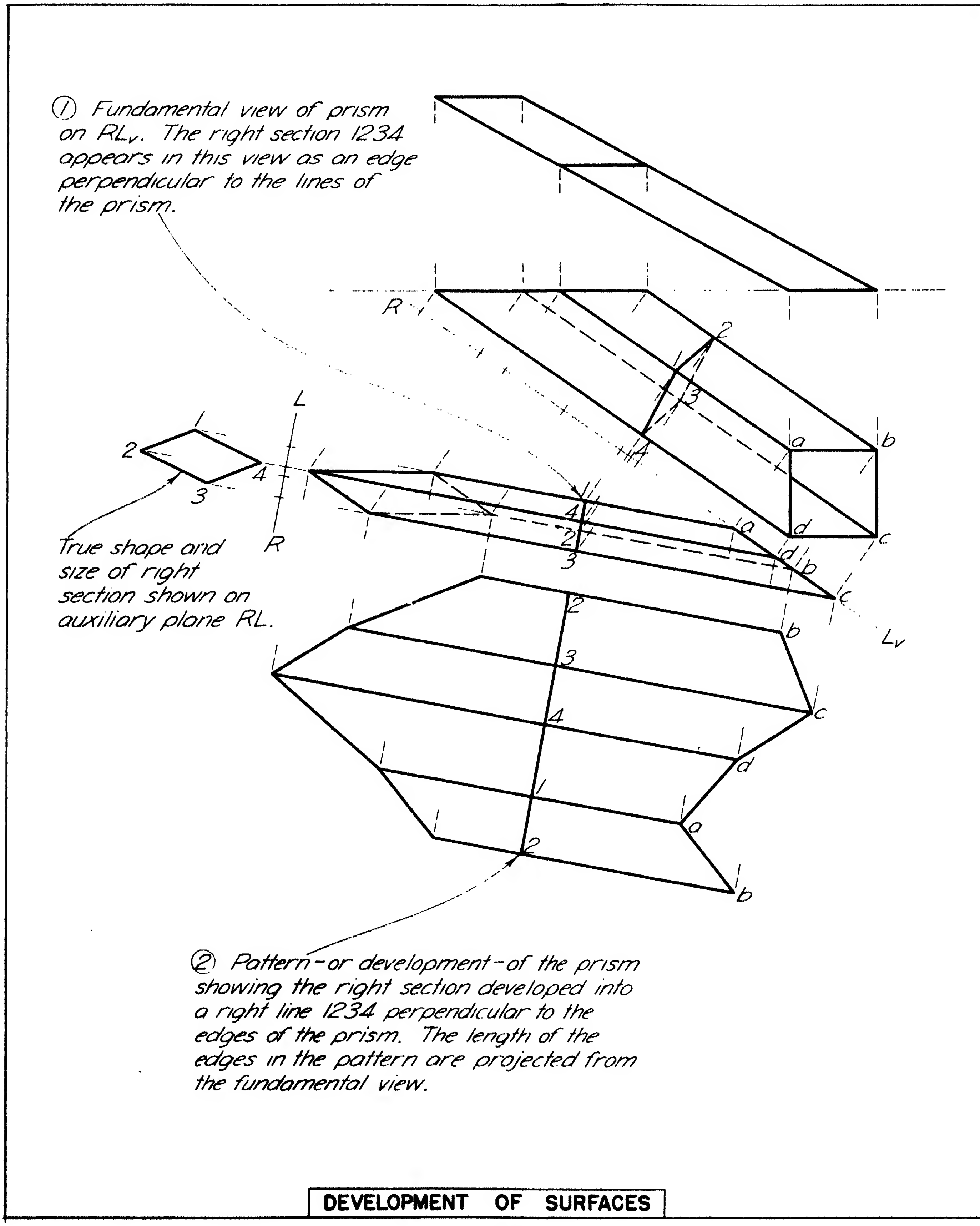


Figure 51
Essential Principle 17.

66. **EP17** (*cont.*). **To lay out a pattern for a surface.**

Cylindrical Elbow Section. Figure 52. Since the right section will develop into a straight line perpendicular to the elements of the cylinder:

1. Divide the right section (use the fundamental view) into equal divisions.
2. Lay off a straight line of indefinite length; divide this line into equal and corresponding divisions according to the right section as at 1 to 8 inclusive; erect perpendiculars to the developed right section at these points 1 to 8 inclusive; by projection from the fundamental view lay off the length of these perpendicular elements; construct by any convenient method a second half of the pattern equal to the half shown from 1 to 8 and draw a smooth curve through the developed ends of the truncated elements. This will be a pattern for one section of the cylindrical elbow.
3. The partial drawing indicates how the same right section may be used in developing the middle section of the elbow.

For convenience in drawing a pattern of this type the drawing paper should be shifted on the drawing board so that the developed line of right section is parallel to the T-square. In this way, distances may be transferred from the fundamental view to the pattern with a minimum of manipulation and with greater accuracy.

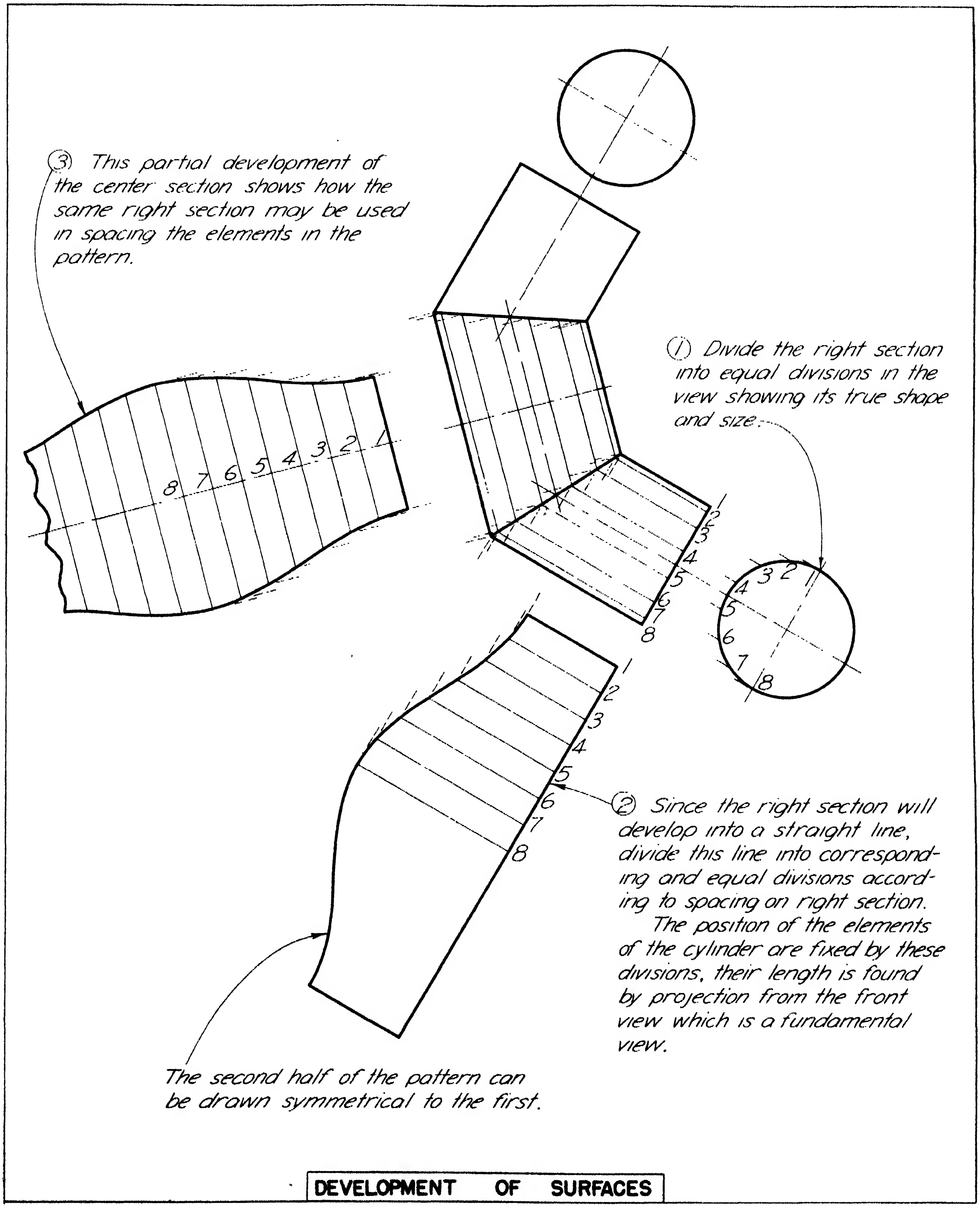


FIGURE 52
Essential Principle 17 (*cont.*).

67. **EP17** (*cont.*). **To lay out a pattern for a surface.**

Right Cone. Figure 53. To develop a pattern for the right cone between its circular base and its intersection with the open prism:

1. Find the true shape and size of the circular base, and divide it as shown. (Although usually such divisions may be most conveniently transferred to the pattern when made equal, cases arise where it is more important to have elements of the development pass through critical points. In this case, divisions are not equal and each distance must be transferred by measurement from the right section.)

2. With any convenient point O as a center (on the pattern) describe an indefinite circular arc whose radius is equal to $O1$, the slant height of the cone. Choose at random the position on the paper of short element $O1$, and mark off from 1 the positions of points 3 to 13 inclusive by transfer from the right section.

3. Find on the front view of the cone the positions of all elements from 1 to 13 inclusive. By revolution into the limiting element (either $O1$ or $O13$, front view) find the true length of all distances on these elements from O to the curve of intersection (ox on element $o5$ is shown in true length at ox_1 and indicates the method). Lay off these distances from O to the curve on the corresponding element position in the pattern and construct a similar half of the pattern beyond element $O13$.

When all points on the curve of intersection have been located on the pattern, a smooth developed curve of intersection may be drawn through these points and the pattern will be outlined as shown.

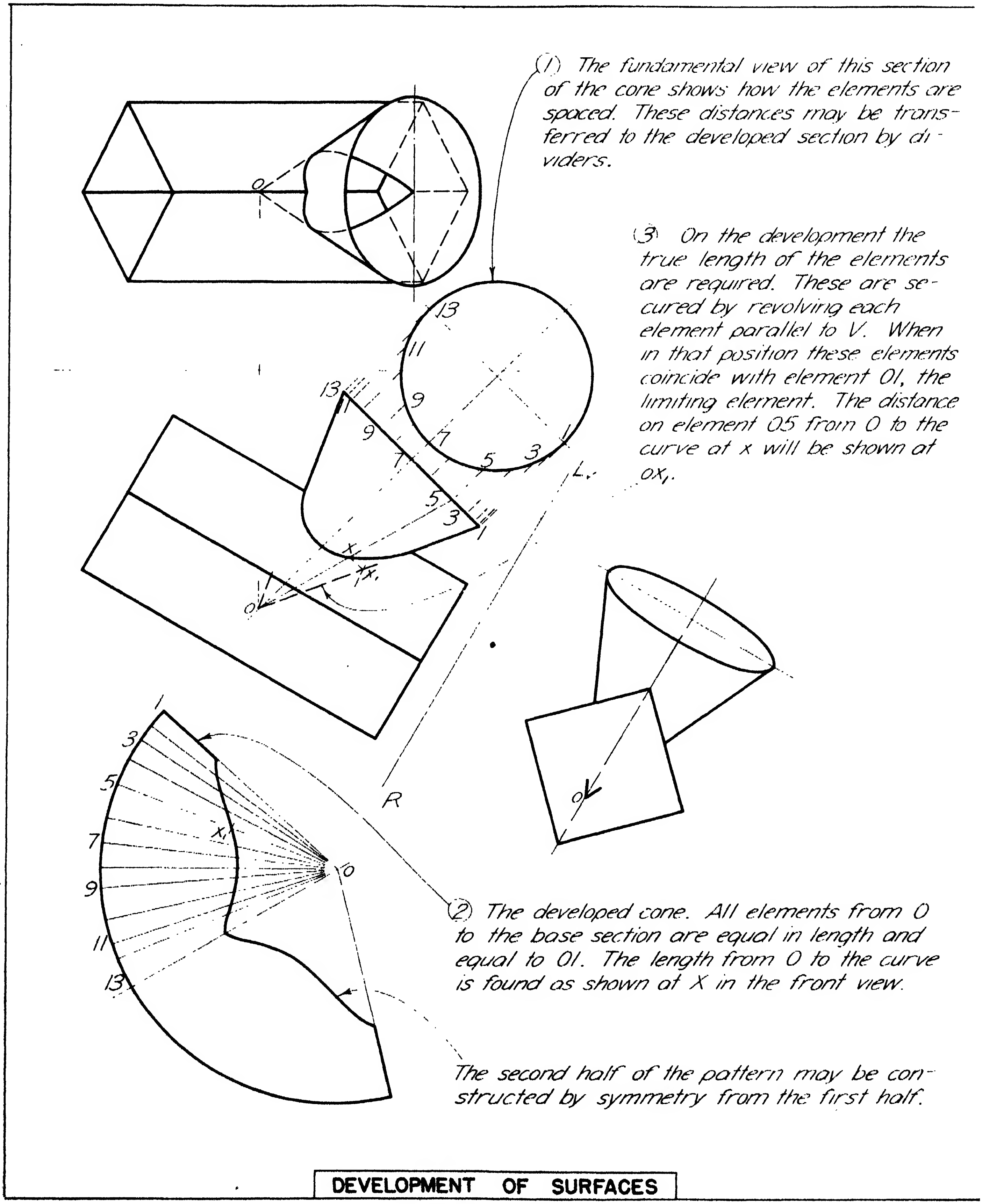


FIGURE 53

Essential Principle 17 (*cont.*).

68. **EP17** (*cont.*). **To lay out a pattern for a surface.**

Surfaces which can be developed, or flattened out onto a plane surface, are plane surfaces and single-curved surfaces. By approximate methods, however, patterns may be constructed for surfaces which are not developable. The skill of the draftsman in dividing the non-developable surface up into segments which are developable and of arranging these into a pattern will determine how closely the surface formed from his patterns approximates the original.

The most satisfactory method of laying out patterns for non-developable surfaces is the method of triangulation. This method consists of covering the surface to be developed with a network of small triangles having a common side and of laying out the true shape, size, and relationship of these triangles into a pattern.

Warped Surface. Figure 54. The section of the "corner hopper" between its flat sides is a portion of a warped cone.

1. Its surface is divided up by a network of small triangles as *A*12, *A*2*B*, *B*23, etc., each being small enough practically to coincide with the surface.

2. Beginning at *A*1, the small triangle *A*12 is constructed on a chosen center line for the pattern. The true lengths of *A*1, of *A*2 must be found by revolution, Article 41, Fig. 30. After triangle *A*12 is constructed, triangle *A*2*B* is constructed with side *A*2 in common with the first triangle. The distances *ab* and 12 are shown in the top view; the lengths *A*1, *A*2, *B*2, etc., must be found by revolution. The method of doing this is indicated for *A*2 and *B*3. All the triangles on the surface having been constructed in their true shape, size, and relation, a pattern will appear as shown. The flat sides of the hopper are developed as planes.

The basic principles of template making have an application of interest to engineers. It is obvious when concrete is to be poured in the shape of a single curved surface, the forms can be made up of very narrow strips of form lumber which correspond closely enough for practical purposes to the linear elements, or rulings, of the surface. It should also be obvious from a study of Fig. 54 that surfaces similar in character may be closely approximated in concrete by building forms made up of lumber cut so that sections are like *ab*21 and *bc*32 and are built up adjacent to each other.

The use of sheet-metal forms is not uncommon in shaping concrete to unusual designs and such forms may be developed by the methods outlined for pattern making.

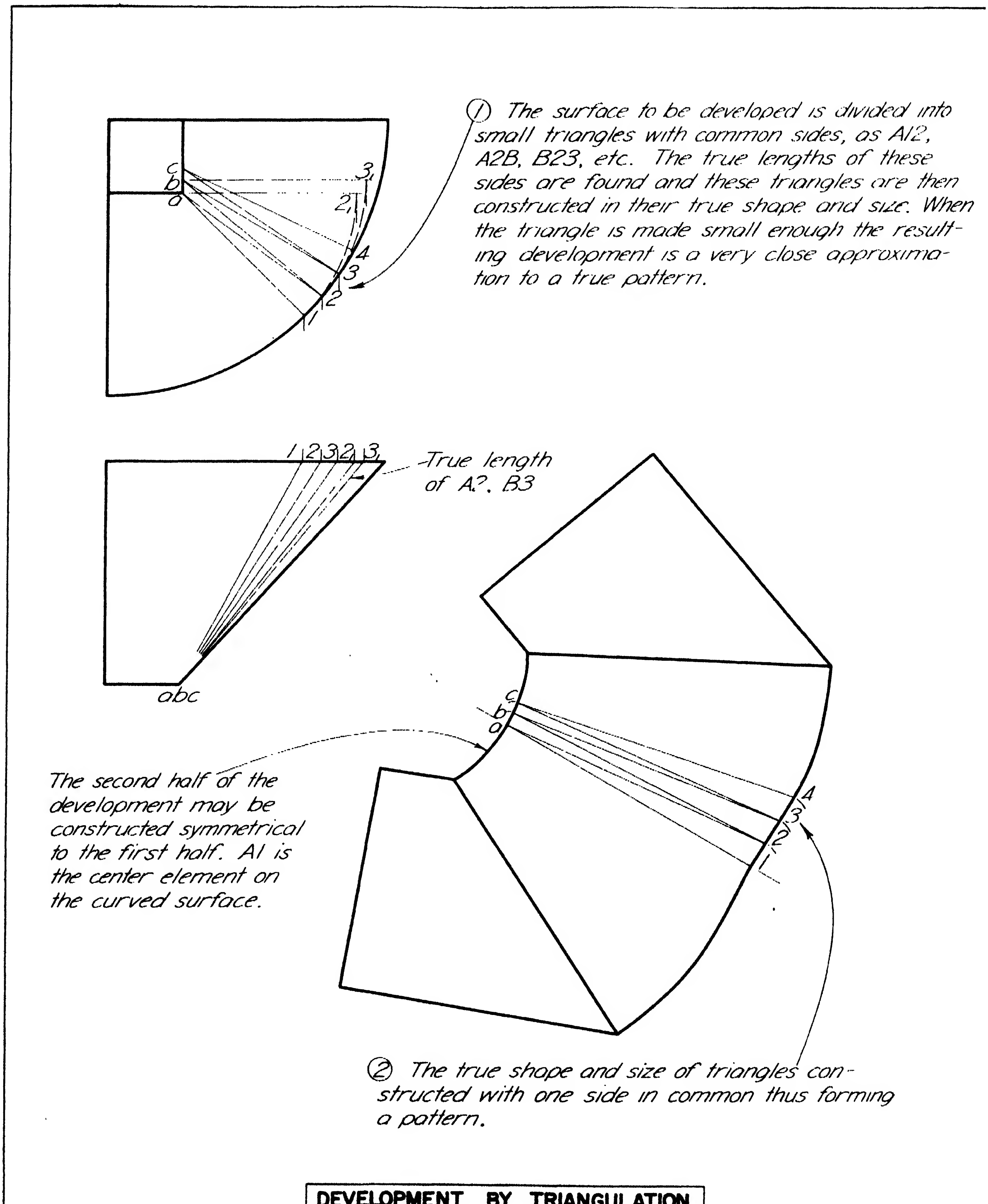


FIGURE 54
Essential Principle 17 (*cont.*).

69. EP17 (*cont.*). **To lay out a pattern for a surface.**

Surfaces of double curvature are not developable. Patterns for such surfaces which will give a satisfactory approximation of the surface may be constructed by dividing the surfaces into zones, or into gores, or by combining zones and gores.

Zones are usually developed as cones, since these developable surfaces can be made to fit the original more closely. Gores are constructed by a method resembling spherical triangulation.

Double-Curved Surface. Figure 55. The drawing represents the approximate method for the development of a spheroid, but the method will apply to spheres and other similar surfaces. To lay out a pattern for a spheroid: (*a*) the surface may be divided into zones each of which may be developed as a right cone; (*b*) the surface may be divided into gores of equal size which may be developed as cylinders, or (*c*) a combination of zones and gores may be used.

1. Zone method. The apex of a right cone must be found which will have limiting elements practically coinciding with the contours of each zone. These cones when developed and joined will form a surface closely approximating the given surface.

2. Gore method. Each gore is assumed to be cylindrical in character, and if the gores are made sufficiently narrow the resulting pattern will closely approximate the given surface.

The several parts of a surface, shaped from patterns made by either of the above methods, must, of course, be formed by shop processes to conform to the curvature of the surface before these several parts are assembled.

Both the *zone* and the *gore* method of laying out a sphere on a flat surface contain the basic elements of map projection, and are used by makers of globe maps. The zone system is a form of polyconic projection, which is but one of more than a dozen forms of map projection to be found in works on cartography.

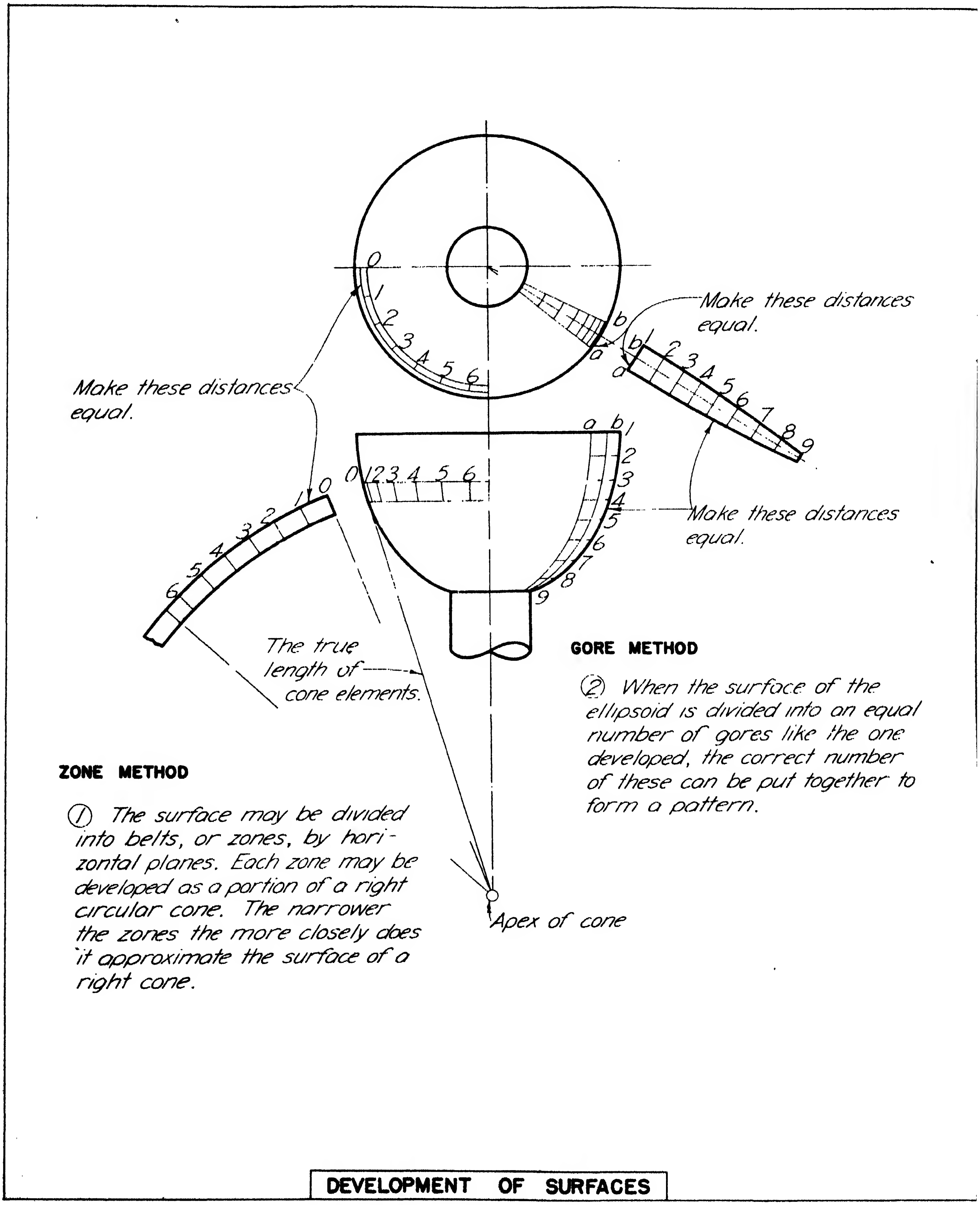
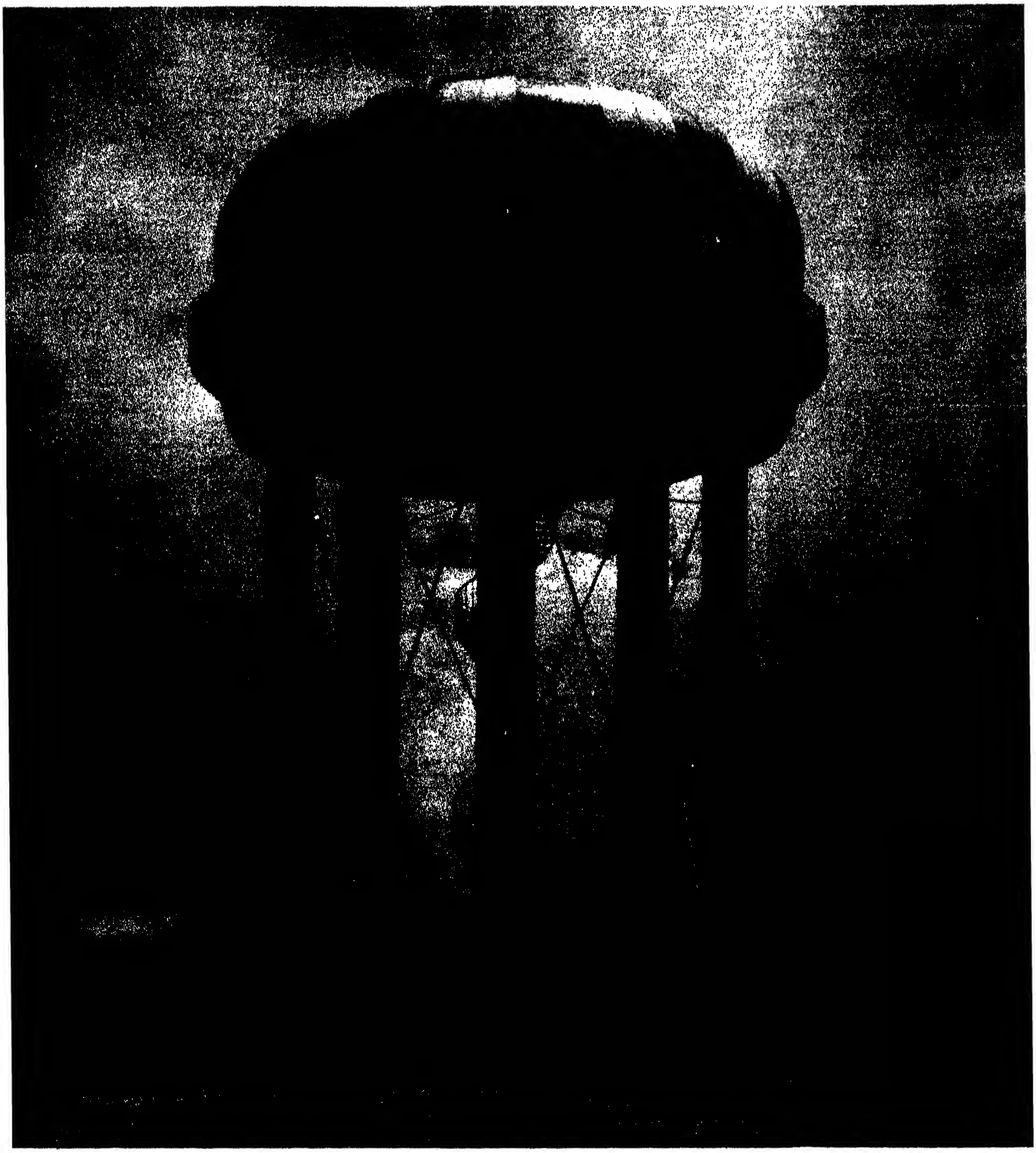


Figure 55
Essential Principle 17 (*cont.*).



In addition to showing an interesting use of cones, cylinders, spheres in an industrial structure, the so-called "spiral" stairway illustrates the use of the helix. The unusual painting on the tank is required because it is located on an airlane.

CHAPTER VI

SCREW SURFACES

70. One of the oldest and most important mechanical devices used by engineers is the screw surface. Its applications are many and include threads for fastenings; cams for transmitting motion; coiled springs; conveyor flights; propellors for ships and airplanes; fan blades for water wheels and pumps; and similar uses.

The basic geometrical element in both design and in representation of screw surfaces is a space curve called the helix, and this curve becomes a controlling element in the generation of such surfaces.

71. *The helix* is a space curve generated by a point moving along and around an axis. If the distance from the axis is uniform (and the curve therefore wrapped around a cylinder) the curve is identified as a *cylindrical* helix; if the distance from the axis varies uniformly (and the curve therefore wrapped around a cone) the curve is identified as a *conical helix.*

The cylindrical helix is the directing element of most of the screw surfaces which have practical application; the conical helix has more limited application. The drawing of these curves and a number of applications are illustrated in the following pages.

72. **To Draw a Cylindrical Helix. Figure 56.** The data required for the construction of a helix are the distance from the axis (the radius of the cylinder upon which the helix is to be wrapped) and the distance and direction the moving point advances in one turn (lead). Given these data, a helix may be drawn:

1. Divide the vertical lead into any convenient number of equal spaces.

2. Divide the top view of the diameter of the cylinder into the same number of spaces.

3. For each one of these divisions of rotation *around* the axis, the generating point on the helix moves one division *along* the axis. By plotting the top and front views of each of these points and connecting them in order, a helix may be plotted.

4. A coiled spring (one turn) based on the same helix is drawn by locating squares equal to a right section of the stock whose centers follow the helix and drawing the edge contour elements.

73. **To Draw a Conical Helix.** The same general principles for drawing a cylindrical helix apply. See Fig. 56.

5. Note especially how the location of the top and front view of point 2 is obtained.

6. A coiled spring made up of round bar stock and coiled on a center line of the same size as the conical helix. The contour outline may be found by locating spheres on the helix as shown.

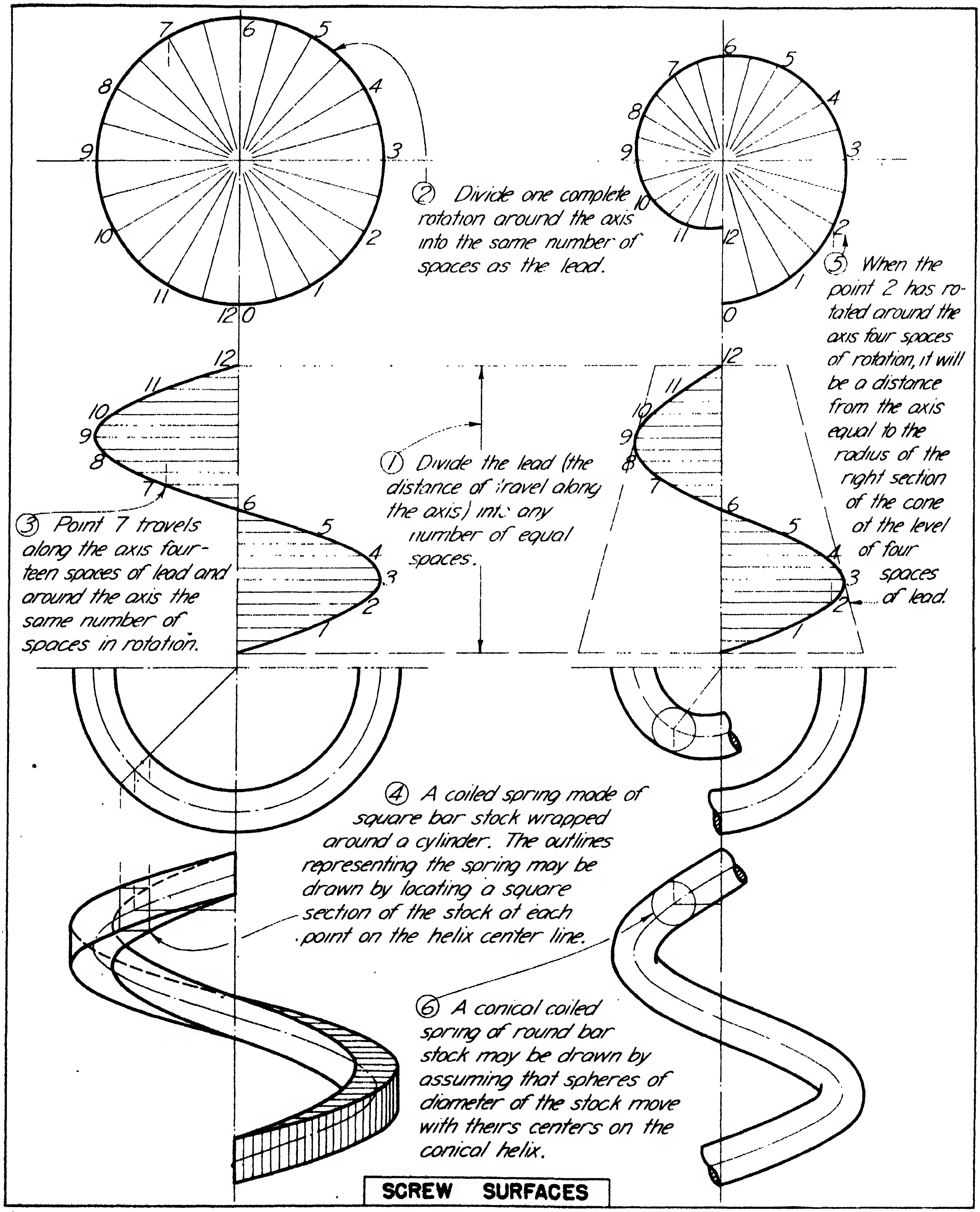


FIGURE 56
The construction of the helix.

74. **EP18. To draw a right helical conveyor flight. Figure 57.** Certain data must be available: as the outside diameter, the diameter of the shaft on which the conveyor is to be mounted, and the lead in order to design a conveyor. Given these data:

1. By dividing one rotation and the lead into the same number of divisions, the front and top views of both the inner and outer helix which bounds the conveyor flight may be drawn as illustrated in Article 72, Fig. 56.

In Fig. 57, the shaft on which the conveyor flights are to be mounted is shown, and one flight (or one complete turn) of the conveyor is drawn.

75. **To lay out a pattern for this conveyor flight. Figure 57.** Such a surface (a right helicoid) is a warped surface and is, therefore, not developable. Practically, however, patterns for such surfaces closely approximating the true surface may be made; when pressed into shape and joined, flights of conveyors so built are close enough to the theoretical surface to be acceptable.

2. Divide the surface of a conveyor flight into small triangles. In Fig. 57, the space between each two successive positions of the elements has been made into two triangles by drawing a diagonal such as 89.

3. Since each area so divided is equal, and since the elements are shown in true length and are equal, the true length of one diagonal only is necessary to proceed with the development. The true length of this diagonal may be found by locating its fundamental view as shown.

4. Beginning with element 88, located on the paper in any convenient position where space is available for a pattern, construct the triangles into which the flight has been divided in their true shape and size and with a diagonal in common. When all these triangles have been drawn the approximate development of the helical conveyor flight will be outlined.

5. It is to be noted that the true spacing of the elements on the inner and outer curves of the pattern is to be taken from the triangles which show the rectification of these curves giving both true length and true spacing.

Since the curvature of the inner and outer curve on the conveyor pattern is fixed by the constant distance between them, the ends of the pattern may overlap. In such cases, a portion—say three-fourths—of one flight may be used as a pattern. When these partial flights are joined, they make a continuous conveyor whose joints are staggered around the axis.

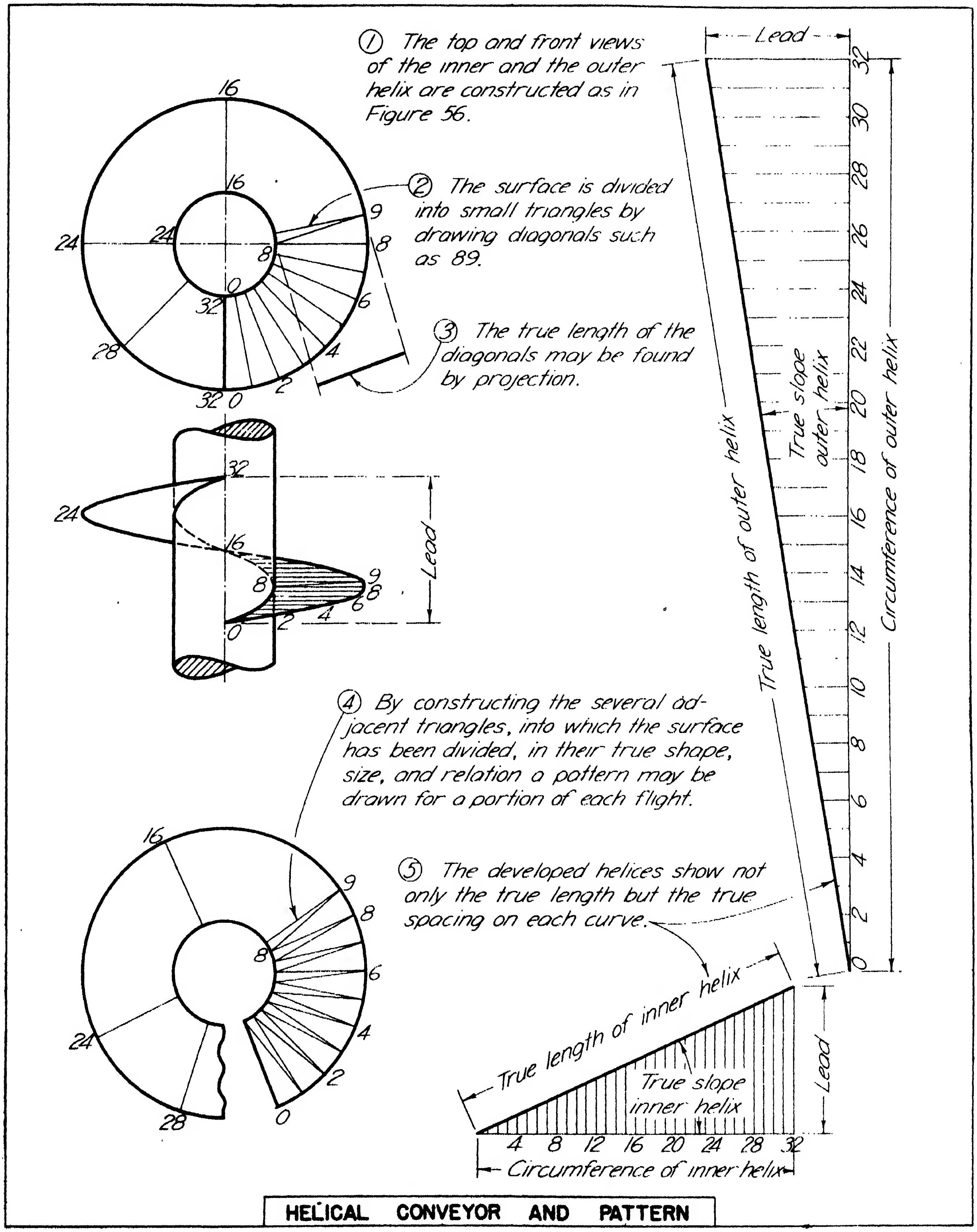


Figure 57

Essential Principle 18.

76. **EP19. To draw a convolute conveyor flight. Figure 58.** As in the right helical conveyor, the shaft diameter, the conveyor tube diameter, and the lead must be known. For the sake of comparisons, these data are the same as in the helical conveyor. To draw the conveyor:

1. Draw one complete turn of the inner—or shaft—helix. To locate a point on the outer helix: draw the plan view of an element tangent at O to the shaft helix. Where the element cuts the plan view of the conveyor tube at O is the other end of the generating element.

To find the front view of O: draw the front view of OO, a line through O on the shaft helix which slopes at Θ, the slope angle, of the shaft helix. (This slope angle Θ is found by graphical use of a right triangle having the lead for the altitude and the circumference of the shaft as a base. The hypotenuse of this right triangle is the true slope Θ and the true length of the inner or shaft helix.

With this point O on the outer helix located, the outer helix may now be drawn as in Article 72. The inner and outer helix, together with the two limiting positions of the generating line OO, define the contour of one flight.

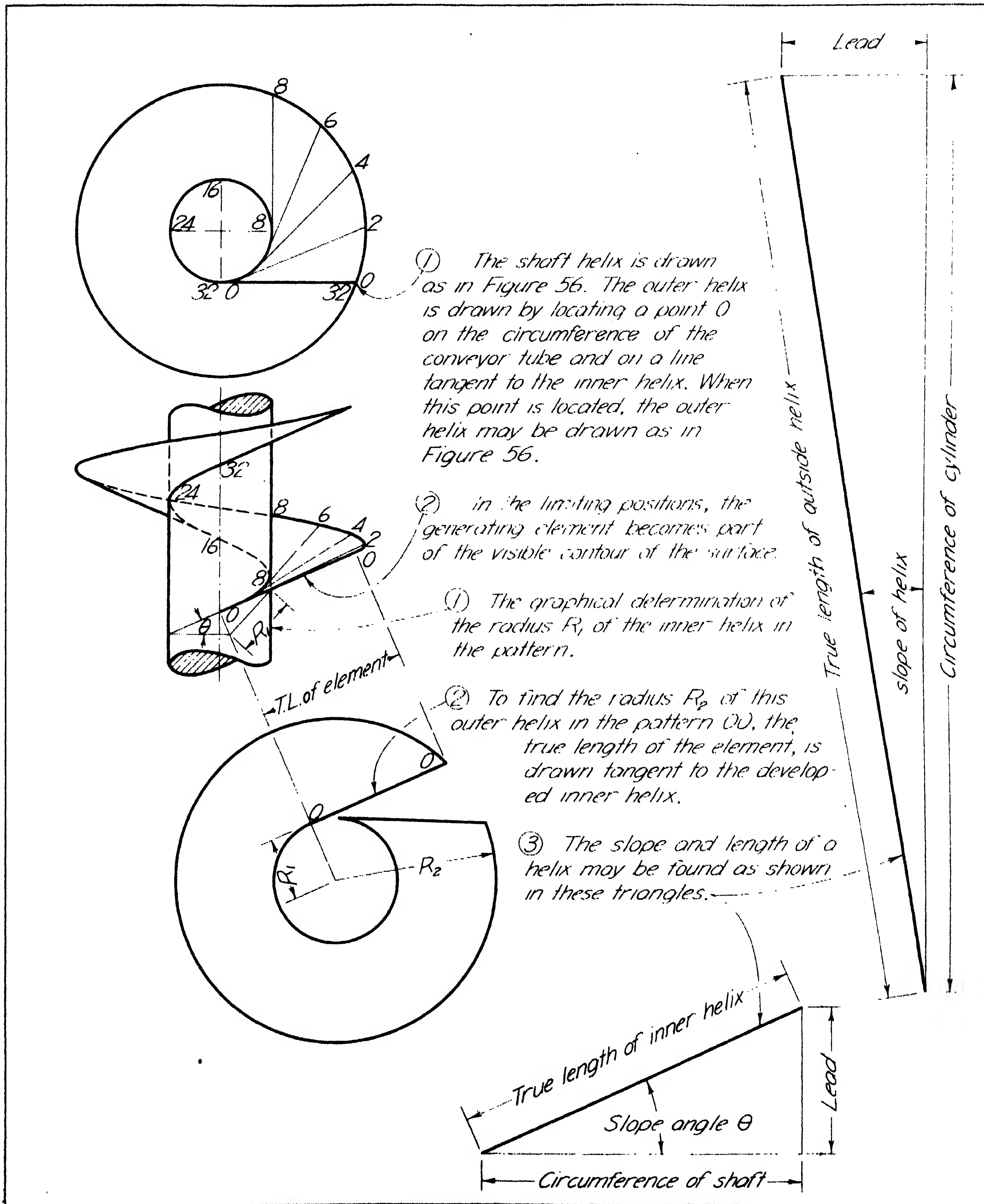
2. Note that the limiting (tangent) position of the element is visible and appears in true length and slope.

77. **To develop a pattern for this conveyor flight. Figure 58.** Since a convolute is generated by a line moving tangent to a helix, it is a single-curved surface and therefore developable. A pattern for one flight of a convolute conveyor develops into a partial circular disk. To find the radii of the curves:

1. The radius of the shaft helix, when one flight is developed, becomes the slant height of a cone whose base is the true distance along an element of the convolute between the limiting elements of the shaft and whose slope angle is the slope angle of the shaft helix. This radius is R_1 in Fig. 58.

2. The radius of the outer curve of the pattern is found by drawing an element in its true length tangent to the developed inner helix.

3. The length of the inner and outer curves of the pattern may be found by stepping off on each the true length of these curves as found in the triangles showing the rectification of the helices.



CONVOLUTE CONVEYOR AND DEVELOPMENT

FIGURE 58

Essential Principle 19.

78. **Screw Threads.** The surface of the thread form used on bolts, nuts, and screws is helicoidal and varies in characteristics according to the profile of the tool used to cut the thread. Several standard thread forms, such as V-threads, square threads, Acme threads, and worm threads, are used and are described in detail in textbooks on engineering drawing. Since the representation of threads on bolts and nuts follows the same general principles for all forms, only two common types—the sharp V-thread and the square thread—are illustrated in this book.

The representation of threads in engineering drawings is a highly conventionalized description, and the involved geometry of construction for exact representation is both unusual and unnecessary. Engineering draftsmen should, however, understand the geometry of thread construction and the differences between actual representation, conventionalized representation, and thread symbols.

79. **Sharp V-Thread Representation.** Figure 59 illustrates a two-pitch (two threads in the lead distance) V-thread designed to emphasize features not usually described in such thread representation. The thread is cut into a cylinder by a tool shaped as a V, the angle at the vertex being 60°, and the depth of the cut being adjusted so that exactly two threads fit into each lead distance.

The large helix, representing the ridge of the thread, on the surface of the cylinder, and the small helix, representing the path of the point of the tool, are drawn as indicated on the drawing and as described in Fig. 56. To find the edges made by the cutting tool as it enters the cylinder and finally reaches its full cutting depth, and the edges made by the cutting tool as it cuts into the conical chamfered end is the unusual part of this problem.

The assumption is made that the cutting tool leaves the cylinder at *O*, having been uniformly withdrawn from full cutting depth at the limiting meridian position *VIII*. The width of the cut for each position of the tool is shown in the diagram, and by plotting these positions the edges on the surface of the cylinder and the helix cut by the point of the tool may be plotted. The end view of this path is a spiral shown in dotted lines.

In like fashion the curve showing the edge made by the cutting tool as it cuts through the conical chamfered end may be found: the intersection of the cutting edge of the tool with the appropriate element of the cone must be found for each successive position of the tool. The construction at point *e* is illustrative.

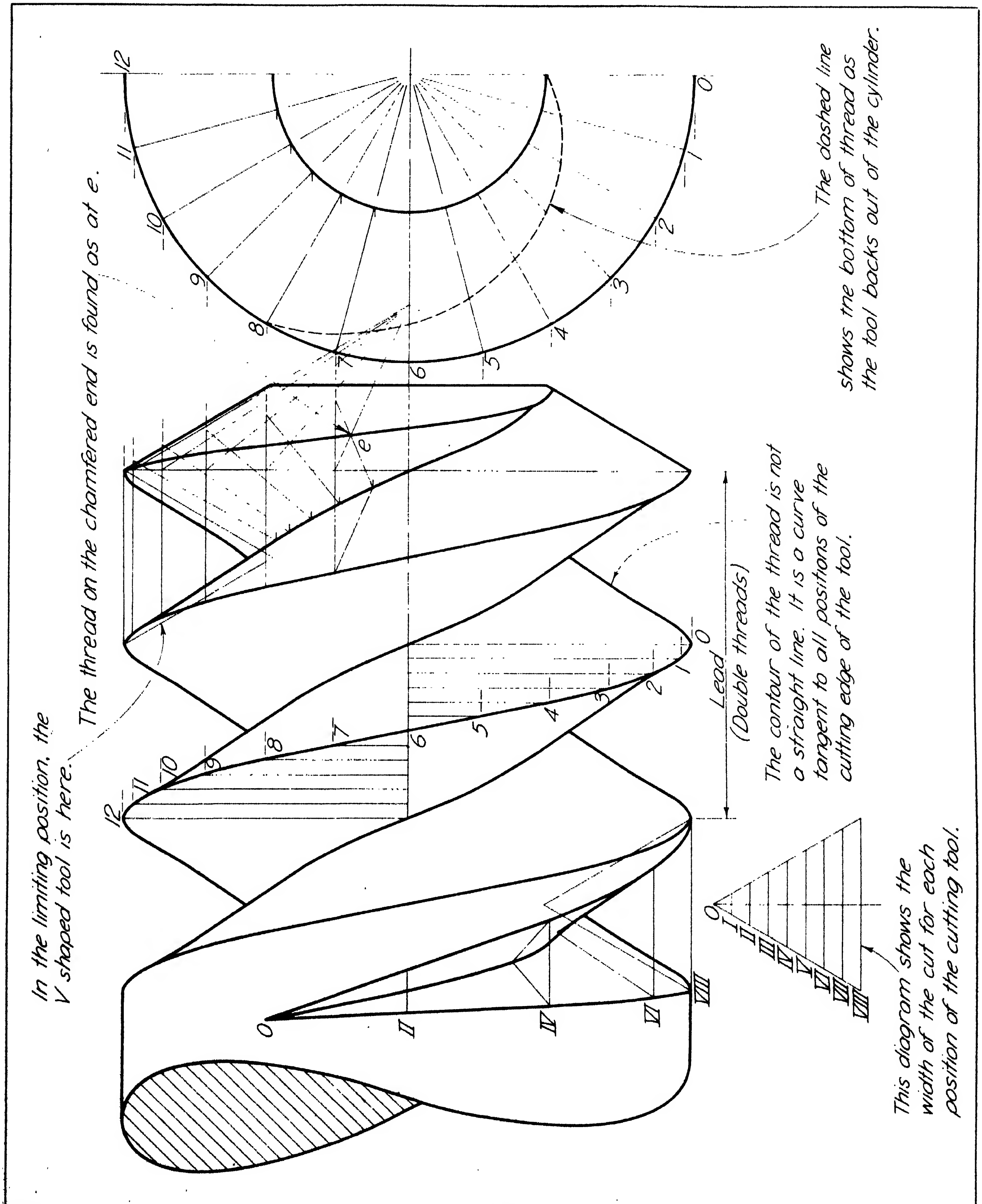


Figure 59

Representation of V-thread screw.

80. **Conventionalized Thread Representation.** Mention was made in Article 79 that thread representation when not symbolized is conventionalized. True thread representation is seldom drawn because conventional representation is usually adequate and thread symbols are universally understood. Moreover, threads on drawings are generally standard, and are so minute that exact representation would be unnecessary.

This is illustrated in Fig. 60, where exact representation may be compared with conventionalized representation. The thread representation nearest the binding edge of the text is an exact representation drawn as described in Article 79. The nut is shown in section. The other thread representation is the same thread conventionalized both by the helix convention and by the straight-line convention. The nut is shown in section by the straight-line convention.

81. **To draw a conventional V-thread representation.** Lay off the pitch distance on the limiting elements of the cylinder on which the thread is to be cut, being careful to allow for an advance of one-half the lead in the one-half turn. Draw the V contours of the threads in these limiting positions, noting that the ridges are opposite the valleys. It is also to be noted that the pitch is equal to the lead for single threads, one-half the lead for double threads, and so on. This is important as it influences the size of the thread profile in the limiting position.

For the helix convention: Draw the ridge helix and the valley helix as shown in Fig. 56. Note carefully how right-hand and left-hand threads affect the slope of this curve. The threads in Fig. 60 are right hand. (This means that a nut turned in a right-hand direction will screw on the thread.) One end view of the two helices will suffice for the construction of the entire representation. A simpler device for large threads is to construct one-half a turn for each helix and make a template of thin celluloid for the drawing of the threads.

For the straight-line convention: Connect the ridges and valleys by straight lines as shown in the figure. When nuts, or internal threads, are drawn in section it is to be noted that the ridge and valley lines slope in the opposite direction from those on the screw. This is because the threads on the back are being represented.

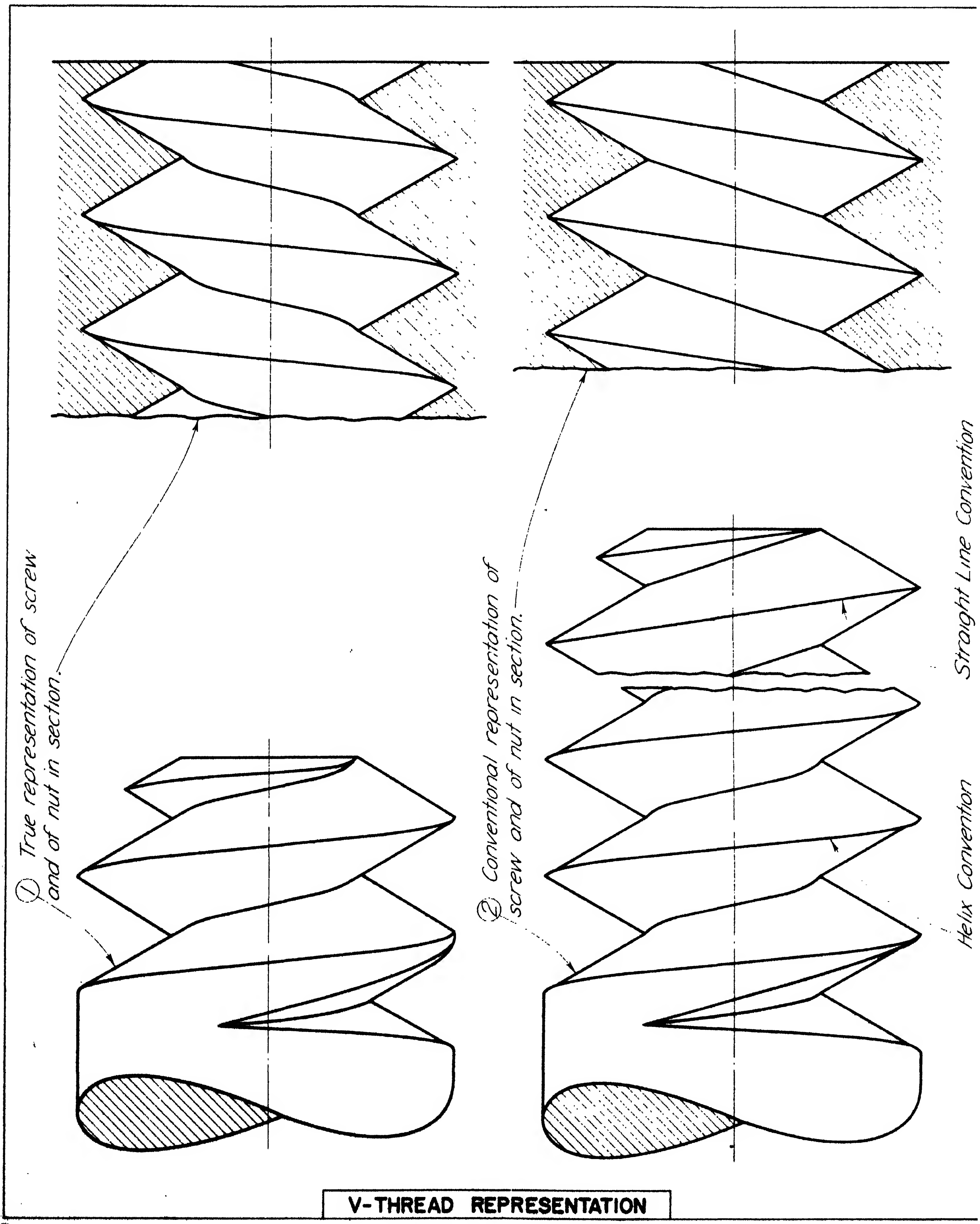


FIGURE 60

Conventional representation of V-threads

82. **Square Threads.** Threads of the square type are usually larger than those of the V-thread type which are used for fastenings. It is, therefore, not uncommon to find square threads, Acme threads, and worm threads shown on drawings as exact representations. When not so drawn they are shown by the straight-line convention. In Fig. 61 a square thread of the same pitch and diameter is shown by the true helix representation and by the straight-line convention. The internal thread is shown in section.

83. **To draw a square thread.** In drawing a square thread one-half the pitch distance is laid off as thread and one-half as space. The limiting elements of the cylinder on which the thread is cut is so divided, care being taken to have spaces on one limiting element opposite threads on the other and to have the correct slope for right-hand or left-hand threads as specified. Since the depth of the thread is one-half the pitch, the diameter at the root of the thread is known. With the outside and root diameters known, and the contours of the threads drawn in the limiting positions, the helices for the thread edges may be drawn as shown in Fig. 61 and as described in Fig. 56.

With one-half of each helix drawn, a template may be made and used for drawing the threads. The use of a template is helpful especially in inking.

If the straight-line convention is to be used, lines connecting the appropriate corners of the thread contour in the limiting positions are drawn with correct slope in place of the helices.

It is to be noted that, in threads of the square type, exact representation is the same as the helix convention.

The internal threads are drawn in the same manner and, since they are shown in section, slope in the opposite way to the threads on the screw.

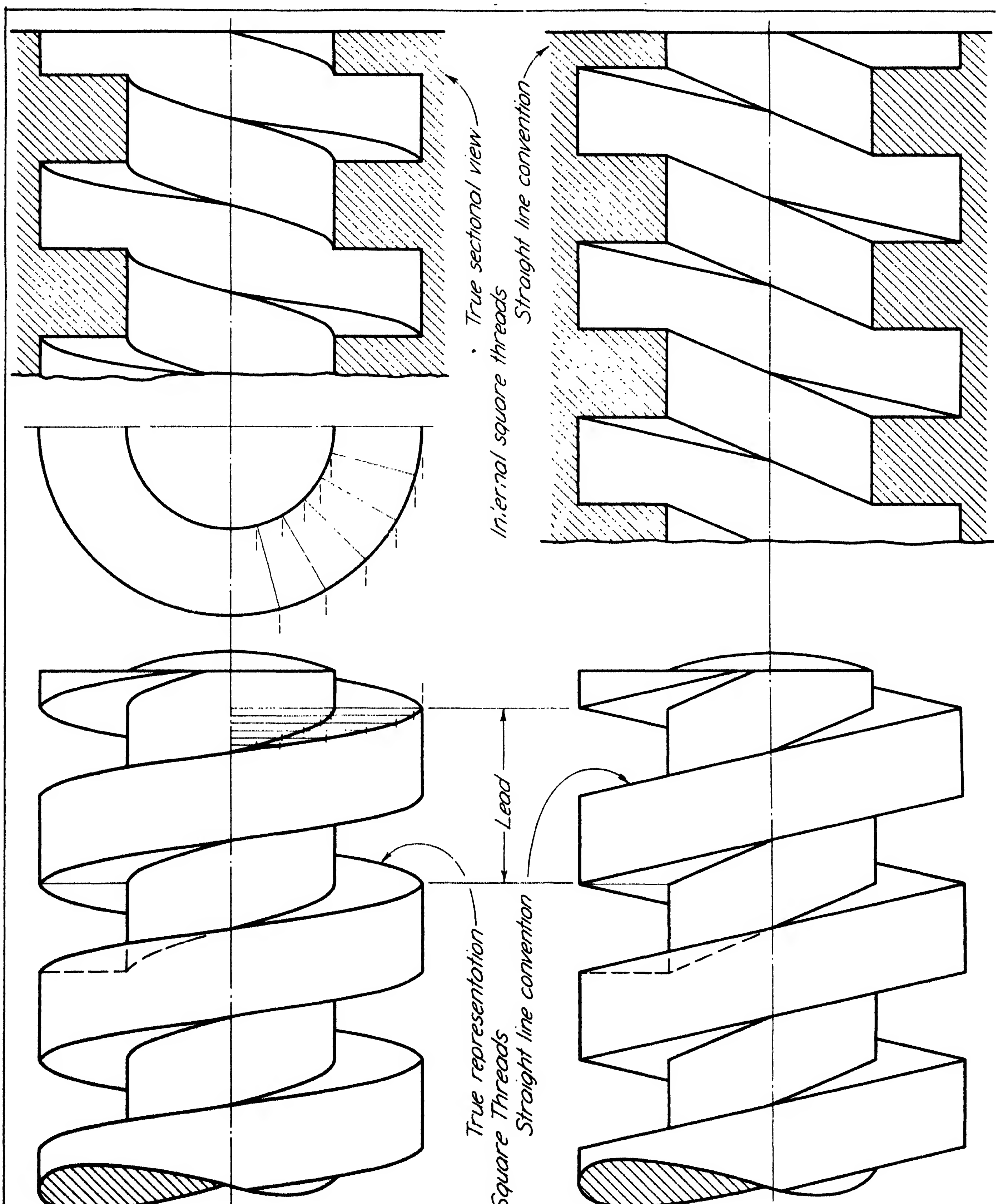


Figure 61
Representation of square threads.